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ANALYTIC SOLUTIONS OF MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY STRETCHING SHEET

Mrs. P. Asvini* **Dr. M. Palanivelrajan****
*Ph.D. (Part-Time) Research Scholar, Department of Mathematics, Dravidian University, Kuppam, Andhra Pradesh State.
**Assistant Professor, Department of Mathematics, Government Arts College, Paramakudi, Tamil Nadu State.

Abstract
The present paper studies the MHD boundary layer flow and heat transfer towards an exponentially stretching sheet. In each case the non-linear momentum equation and the energy equation have been solved using the Adomian decomposition and Homotopy perturbation methods. Comparisons of the applied methods with exact solutions reveal that both methods are tremendously effective.

Key Words: *Boundary Layer Flow, Exponentially Stretching Sheet, MHD, Adomian Decomposition Method and Homotopy Perturbation Method.*

1. Introduction

The study of laminar flow and heat transfer over a stretching sheet in a viscous fluid is of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. Crane [1] investigated the flow caused by the stretching of a sheet.

Many researchers such as Gupta and Gupta [2], Dutta et al. [3], Chen and Char [4], Andersson [5] extended the work of Crane [1] by including the effect of heat and mass transfer analysis under different physical situations. Gupta and Gupta [2] also stressed that realistically, stretching surface is not necessarily continuous. Most of the available literature deals with the study of boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from the fixed origin.

Boundary-layer flows of viscous fluids are of paramount industrial importance and most can be modeled mathematically by systems of nonlinear ordinary differential equations on an unbounded domain. The solutions of these nonlinear two-point boundary value problems are normally obtained by using, for example, the traditional finite difference methods. The ADM [1] can give accurate and efficient analytic solution in the form of a rapidly convergent series without the need for perturbation, linearization and transformation. The simple unified treatment of linear and nonlinear terms contributes to wide significant applications in solving nonlinear differential equations.

In recent years, the application of the Homotopy perturbation method in nonlinear problems has been devoted by scientists and engineers, because this is to continuously deform a simple problem easy to solve into the difficult problem under study. Homotopy techniques were applied to find all roots of nonlinear equations first in [21-28]. Recently, the application of Homotopy theory has become a powerful mathematical tool, when it is successfully coupled with perturbation theory [29-39].

In this work, we use the Homotopy perturbation method and Adomian decomposition method to solve the highly nonlinear ODE to derive an approximate analytical solution.

2. Mathematical Model

Consider the flow of an incompressible viscous electrically conducting fluid past a flat heated sheet coinciding with the plane y= 0. The flow is confined to y> 0. Two equal and opposite forces are applied along the x-axis, so that the wall is stretched

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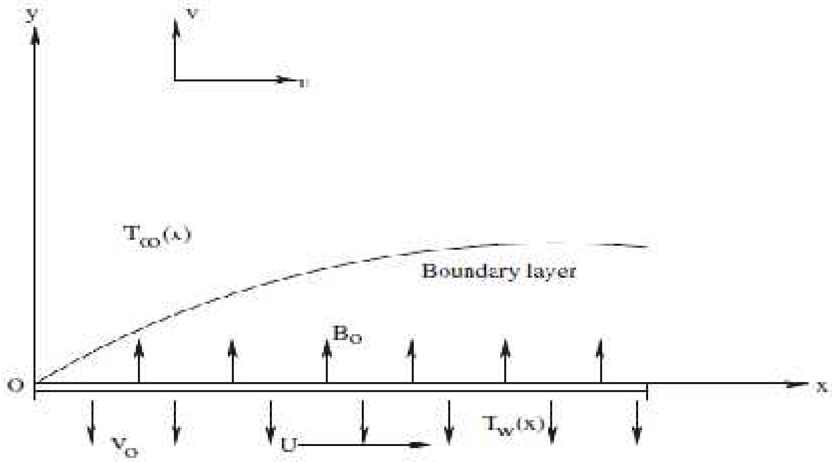

Figure 1 Sketch of the physical problem.

keeping the origin fixed (see Fig. 1). A variable magnetic field $B(x) = B_0 e^{\frac{x}{2L}}$ is applied normal to the sheet, B_0 being a constant [25]. The sheet is of temperature $T_w(x)$ and is embedded in a thermally stratified medium of variable ambient temperature $T_{\infty}(x)$ where $T_w(x) > T_{\infty}(x)$ .It is assumed that $T_w(x) = T_0 + be^{\frac{x}{2L}}, T_{\infty}(x) = T_0 + ce^{\frac{x}{2L}}$ where T_0 is the reference temperature, b >0, $c \geq 0$ are constants.

The continuity, momentum, and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x} + \frac{\partial {}^{3}}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + {}^{3}\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\pm B^2}{\circ}u \quad (2)$$

$$u\frac{\partial T}{\partial x} + {}^{3}\frac{\partial T}{\partial y} = \frac{k}{\circ c_{\circ}}\frac{\partial^2 T}{\partial y^2} \quad (3)$$

where *u* and ³ are the velocity components in the *x* and *y* directions respectively, *T* is the fluid temperature inside the boundary layer, *t* is the time, *k* is the thermal conductivity of the fluid, *v* is the kinematics viscosity, c_p is the specific heat at constant pressure, « is coefficient of fluid viscosity, and ρ is the fluid density. Detail about the MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium can be found in the work of Swati Mukhopadhyay [17].

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by

$$u = U, {}^{3} = -V(x), T = T_w(x) \text{ at } y = 0$$
$$u \to 0, T = T_{\infty}(x) \text{ as } y \to \infty \quad (4)$$

Here $U = U_0 e^{\frac{x}{L}}$ is the stretching velocity, U_0 is reference velocity, V(x) >0 is velocity of suction and V(x)< 0 is velocity of blowing, $V(x) = V_0 e^{\frac{x}{2L}}$ a special type of velocity at the wall is considered. V_0 is the initial strength of suction.

2.2. Method of solution
Introducing the suitable transformations as

$$¦ = \sqrt{\frac{U_0}{2vL}} e^{\frac{x}{2L}} y, \qquad u = U_0 e^{\frac{x}{L}} f\,'(¦)$$

$$³ = -\sqrt{\frac{¬U_0}{2L}} e^{\frac{x}{2L}} \left\{ f(¦) + ¦\, f\,'(¦) \right\}, \frac{T - T_\infty}{T_w - T_0} = ¯(¦) \tag{5}$$

and upon substitution of (5) in Eqs. (2) and (3), the governing equations transform to

$$f\,''' + ff\,'' - 2f\,'^2 - Mf\,' = 0 \tag{6}$$

$$¯\,'' + Pr\left(f¯\,' - f\,'¯ \right) - PrStf\,' = 0 \tag{7}$$

and the boundary conditions take the following form:

$$f\,' = 1,\ f = S,\ ¯ = 1 - St \ \text{at}\ ¦ = 0 \tag{8}$$

And

$$f\,' \to 0,\ ¯ \to 0 \quad \text{as}\ ¦ \to \infty \tag{9}$$

where the prime denotes differentiation with respect to $¦$, $M = \dfrac{2± B_0^2 L}{°U_0}$ is the magnetic parameter, $S = \dfrac{v_0}{\sqrt{\dfrac{³_0}{2L}}} \geq 0$ or ≤ 0 is the suction (or blowing) parameter, $St = \dfrac{c}{b}$ is the stratification parameter and $Pr = \dfrac{«c_{\circ}}{k}$ is the Prandtl number. St > 0 implies a stably stratified environment, while St= 0 corresponds to an unstratified environment.

3. Adomian Decomposition Method for Solution
Usually in the ADM [1], Eqs. (6) and (7) are written in the operator forms

$$L_1 f = L_1\left(-ff\,'' + 2f\,'^2 + Mf\,' \right), L_1 = \frac{d^3}{d\eta^3} \tag{10}$$

$$L_2\theta = -Pr\, L_2\left(\left[f\theta' - f\,'\theta - Stf\,' \right] \right), L_2 = \frac{d^2}{d\eta^2} \tag{11}$$

Applying the inverse operators, $L_1^{-1}(.) = \int_0^\eta \int_0^\eta \int_0^\eta (.) d\eta d\eta d\eta$ and $L_2^{-1}(.) = \int_0^\eta \int_0^\eta (.) d\eta d\eta$ on both sides of (10) and (11) obtained

$$f(\eta) = S + \eta + \frac{\alpha_1 \eta^2}{2} + \int_0^\eta \int_0^\eta \int_0^\eta \left[-N_1(f) + 2N_2(f) + Mf\,' \right] d\eta d\eta d\eta \tag{12}$$

$$\theta(\eta) = 1 - St + \alpha_2 \eta - Pr \int_0^\eta \int_0^\eta \left[N_3(f, \theta) - N_4(f, \theta) - Stf\,' \right] d\eta d\eta \tag{13}$$

Where $\sim_1 = f\,''(0)$ and $\sim_2 = ¯\,'(0)$ are to be determined from the boundary conditions at infinity in (8) and (9). The nonlinear terms, $ff\,'', f\,'^2, f\theta'$ and $f\,'\theta$ can be decomposed as Adomian polynomials $\sum_{j=0}^{\infty} B_j, \sum_{j=0}^{\infty} C_j, \sum_{j=0}^{\infty} D_j$ and $\sum_{j=0}^{\infty} E_j$ as follows

$$N_1(f)=\sum_{j=0}^{\infty} B_j = ff'' \quad (14)$$

$$N_2(f)=\sum_{j=0}^{\infty} C_j = (f')^2 \quad (15)$$

$$N_3(f,\theta)=\sum_{j=0}^{\infty} D_j = f\theta' \quad (16)$$

$$N_4(f,\theta)=\sum_{j=0}^{\infty} E_j = f'\theta \quad (17)$$

Where $B_j(f_0, f_1,..., f_j)$ $C_j(f_0, f_1,..., f_j)$ and $D_j(f_0, f_1,..., f_j, \theta_0, \theta_1,..., \theta_j)$ $E_j(f_0, f_1,..., f_j, \theta_0, \theta_1,..., \theta_j)$ are the so called Adomian polynomials. In the Adomian decomposition method [1] f and θ can be expanded as the infinite series

$$f(\eta)=\sum_{j=0}^{\infty} f_j = f_0 + f_1 + f_2 + ... + f_m + ...$$

$$\theta(\eta)=\sum_{j=0}^{\infty} \theta_j = \theta_0 + \theta_1 + \theta_2 + ... + \theta_m + ... \quad (18)$$

Substituting (14),(15),(16) and (17) into (12) and (13) gives

$$\sum_{j=0}^{\infty} f_j(\eta) = S + \eta + \frac{\alpha_1\eta^2}{2} + \int_0^{\eta}\int_0^{\eta}\int_0^{\eta}\left[-\sum_{j=0}^{\infty} B_j + \sum_{j=0}^{\infty} C_j + M\sum_{j=0}^{\infty}\left(f_j'\right)\right]d\eta d\eta d\eta \quad (19)$$

and

$$\sum_{j=0}^{\infty} \theta_j(\eta) = 1 - St + \alpha_2\eta - Pr\int_0^{\eta}\int_0^{\eta}\left[\sum_{j=0}^{\infty} D_j - \sum_{j=0}^{\infty} E_j - St\sum_{j=0}^{\infty}(f'_j)\right]d\eta d\eta \quad (23)$$

Hence, the individual terms of the Adomian series solution of the equation (6)–(8) are provided below by the simple recursive algorithm

$$f_0(\eta) = S + \eta + \frac{\alpha_1\eta^2}{2} \quad (24)$$

$$\theta_0(\eta) = 1 - St + \alpha_2\eta \quad (25)$$

$$f_{j+1}(\eta) = \int_0^{\eta}\int_0^{\eta}\int_0^{\eta}\left[-B_j + C_j + Mf_j'\right]d\eta d\eta d\eta \quad (26)$$

$$\theta_{j+1}(\eta) = -Pr\int_0^{\eta}\int_0^{\eta}\left[D_j - E_j - Stf_j'\right]d\eta d\eta \quad (27)$$

For the numerical computation, we take the m-term approximation of $f(\eta)$ and $\theta(\eta)$ as $\phi_m(\eta)=\sum_{j=0}^{m-1} f_j(\eta)$ and

$$\omega_m(\eta)=\sum_{j=0}^{m-1} \theta_j(\eta)$$

4. Results and discussion

The recursive algorithm (26) and (27) is coded in MATLAB. The undetermined values of $\sim_1 = f''(0)$ and $\sim_2 = ^{-}{}'(0)$ are computed using the boundary conditions at infinity in (8) and (9). The difficulty at infinity is tackled by applying the diagonal Padé approximants [6], that approximate $f'(\eta)$ and $\theta(\eta)$ using $\phi'_{15}(\eta)$ and $\omega_{15}(\eta)$ applying infinity to the diagonal Padé approximants [N/N] that approximates $f'(\eta)$ and $\theta(\eta)$ ranging value of N from 2 to 10 provides a two by two system of non linear algebraic equation, then obtained nonlinear system are solved by employing Newton Raphson method. The numerical results of $\sim_1 = f''(0)$ and $\sim_2 = ¯'(0)$ obtained are shown in the following Tables.

Table 1

Numerical values of $-f''(0)$ and $-¯'(0)$ using ADM and HPM with Padé approximants of $\phi'_{10}(\eta)$ and $\omega_{10}(\eta)$ for various of S with M=0.1, St=0.2 and Pr=0.7.

$\sim_1$ and $\sim_2$	ADM-Padé of [6/6]			HPM-Padé of [6/6]		
	S=0	S=0.5	S=1	S=0	S=0.5	S=1
$-f''(0)$	1.3274	1.6019	1.9898	1.3245	1.6045	1.9898
$-¯'(0)$	0.48017	0.5956	0.75127	0.48251	0.5966	0.75527

Table 2

Numerical values of $-f''(0)$ and $¯'(0)$ using ADM and HPM with Padé approximants of $\phi'_{10}(\eta)$ and $\omega_{10}(\eta)$ for various of St at S=0,M=0.1 and Pr=0.7.

$\sim_1$ and $\sim_2$	ADM-Padé of [6/6]				HPM-Padé of [6/6]			
	St=0	St=0.2	St=0.4	St=0.6	St=0	St=0.2	St=0.4	St=0.6
$-f''(0)$	1.3164	1.3274	1.3164	1.3274	1.3164	1.3274	1.3164	1.3274
$¯'(0)$	1.3408	0.98017	1.1927	1.0235	1.3474	0.98021	1.1925	1.0238

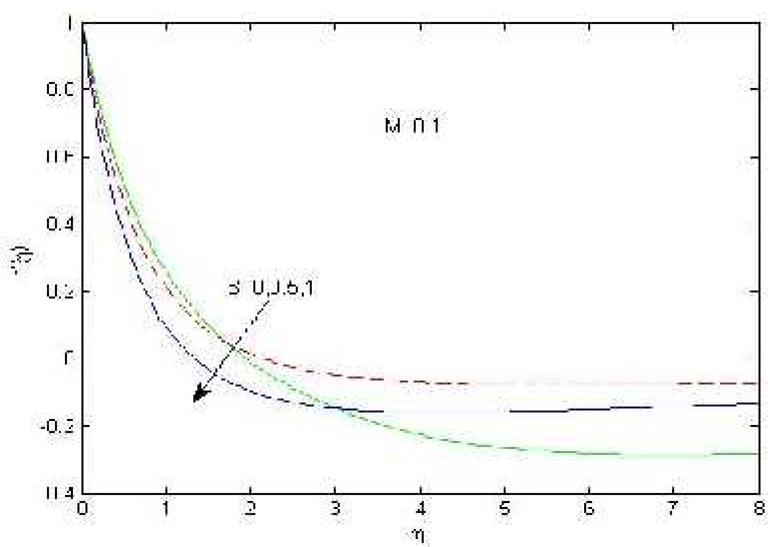

Fig 2 Variation of velocity Profile $f'(¦)$ with ¦ for several values of suction parameter S using $¨'_{10[6/6]}$

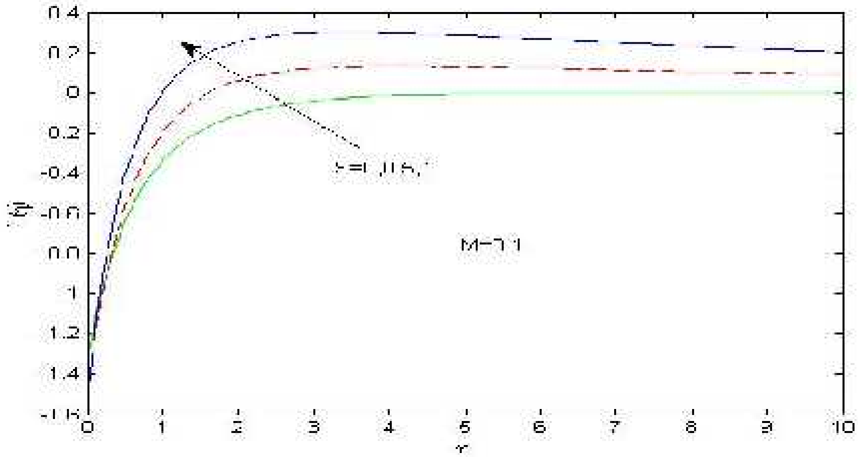

Fig 3 Variation of shear stress $f''(¦)$ with ¦ for several values of suction parameter S Using ¨ "$_{10[6/6]}$.

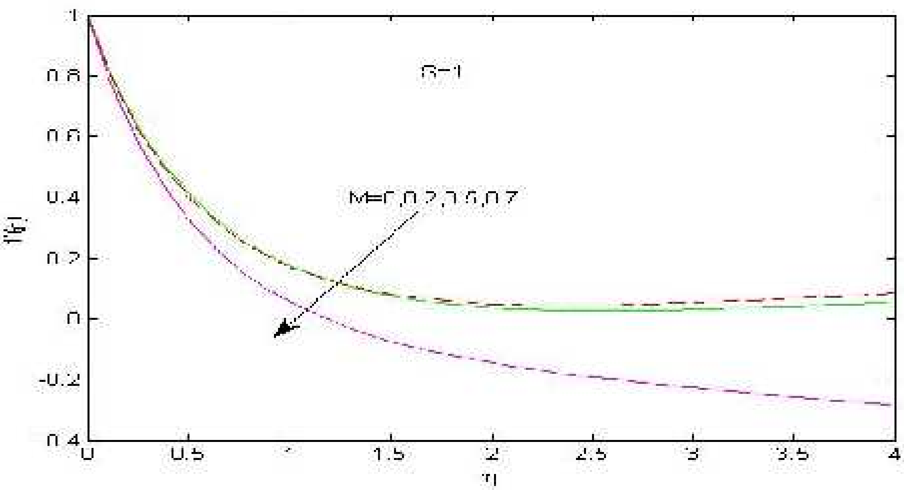

Fig 4 Variation of velocity Profile $f''(¦)$ with ¦ for several values of magnetic parameter M using ¨ "$_{10[6/6]}$.

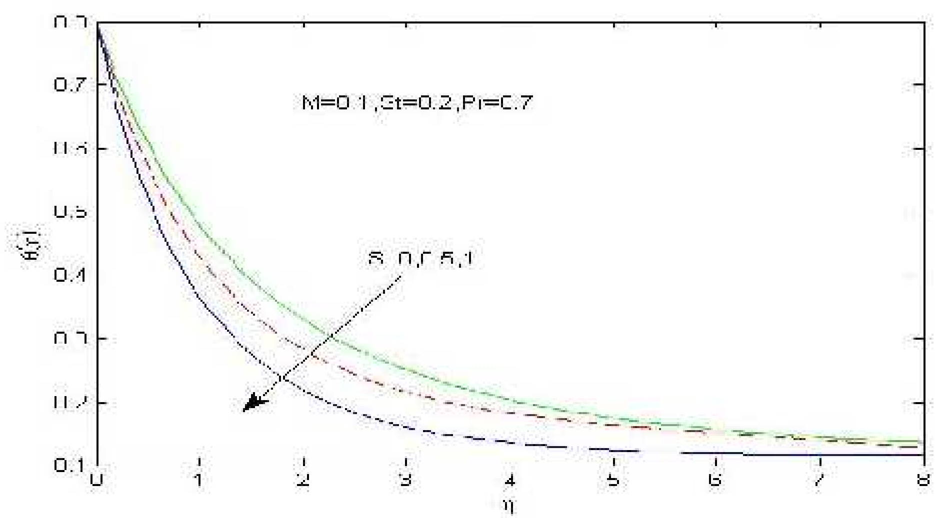

Fig 5 Variation of temperature Profile for ¯(¦) with ¦ for several values of suction parameter S using $\mu_{15[10/10]}$.

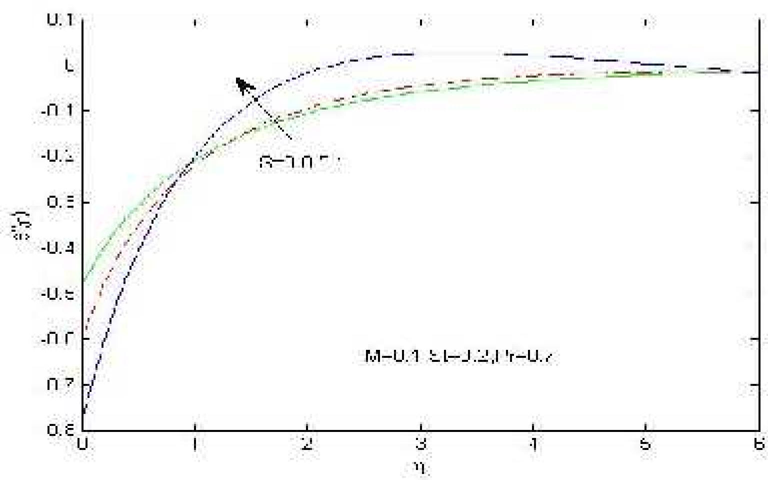

Fig 6 Variation of temperature gradient ¯'(¦) with ¦ for several values of suction parameter S using $\mu_{15[10/10]}$.

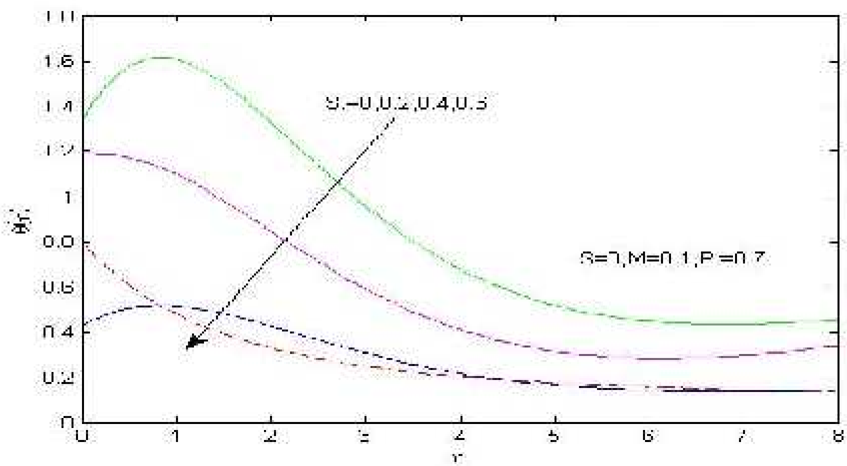

Fig 7 Variation of temperature ¯(¦) with ¦ for several values of St using $\mu_{15[10/10]}$

Figs. 2 and 3 depict the effects of suction parameter S on velocity and shear stress profiles, respectively, for exponentially stretching sheet. It is observed that velocity decreases significantly with increasing suction parameter. From Fig. 4, it is very clear that the shear stress decreases initially with the suction parameter S, but shear stress increases significantly after a certain distance ¦ from the sheet. It is observed that, while considering the wall suction (S>0), this causes a decrease in the boundary layer thickness and the velocity field is reduced. S=0 represents the case of non-porous stretching sheet. Fig. 4 represents the velocity profiles for the variation of magnetic parameter M. With increasing values of M, fluid velocity is found to decrease. The rate of transport decreases with the increase in M because the Lorentz force which opposes the motion of fluid increases with the increase in M.

Figs. 5 and 6 represent the temperature and temperature gradient profiles for variable suction parameter S. It is seen that temperature decreases with increasing suction parameter. The temperature gradient decreases initially with the suction parameter S, but it increases after a certain distance ¦ from the sheet.

In Fig. 7 we represented the effects of thermal stratification parameter (St) on temperature and temperature gradient profiles in the absence and presence of suction at the boundary. Temperature profiles ¯(¦) for different values of the stratification parameter (St) in the absence of suction.

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5. Conclusion

The present study obtains the Adomian decomposition and Homotopy perturbation methods of solutions for steady MHD boundary layer flow and heat transfer over an exponentially stretching sheet in presence of suction. The effect of suction as well as magnetic parameter on a viscous incompressible fluid is to suppress the velocity field which in turn causes the enhancement of the skin-friction coefficient. Rate of transport is reduced with the increasing magnetic field. The temperature decreases with increasing values of the stratification parameter.

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