



PREDICTING DAILY RETURNS OF STOCK INDICES USING FUZZY LINEAR REGRESSION

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Abstract

Forecasting of stock prices and indices is a difficult task. Fuzzy time series models have been applied with reasonable success in financial time series data. However the forecasting accuracies achieved are not sufficient to guarantee positive returns on investing. Neuro fuzzy methods of high order yield better accuracy but carry the risk of over fitting. This paper presents a simple fuzzy linear regression based method for forecasting daily returns. Trapezoidal membership functions are used to represent the fuzzy sets. Empirical results on stock indices indicate that the proposed approach performs better than naive forecasting and neuro-fuzzy solutions in terms of profitability.

Keywords: Fuzzy Time Series, Forecasting, Stock Index, Daily Returns.

1. Introduction

Chaotic time series such as stock prices and indices are inherently complex. The underlying dynamics driving their evolution undergo sudden as well as periodic changes. Nonlinear forecasting methods are required for successful forecasting in such cases [1]. Some of the nonlinear forecasting tools are Neural networks [2], Genetic Algorithms [3], Particle Swarm Optimization [4], hybrid models [5], support vector machines [6, 7], wavelet decomposition [8] and fuzzy logic [9]. Fuzzy time series (FTS) models have received a lot of attention in the last decade as a promising forecasting and analysis tool. The high order FTS models exhibit better accuracy. However for successful forecasting of financial time series, the accuracy must be high enough to make profitable investment decisions. This aspect is not widely reported by forecasting studies. The prediction must result in better profits than simple buy and hold strategy over a long period of time in order to be significant. Overfitting is a serious problem affecting higher order models. Interval length selection and dealing with noisy data are also leading issues in FTS. Improving the accuracy has to be done while avoiding overfitting where the method memorizes the training data without learning generalized rules. The next section presents a brief literature survey of related methods.

2. Related Methods

FTS was proposed by Song and Chissom [10] to forecast the number of enrolments in Alabama University. It introduced a forecasting approach where historical data are expressed as linguistic variables. Subsequent research efforts were focused on the establishment of fuzzy relationships and interval partitions. Some notable research works are Hwang et al. Sullivan and Woodall and Tsaur and Woodall Chen and Chung applied genetic algorithm (GA) to determine interval partitions. Kuo et al. utilized particle swarm optimization (PSO) in Yu introduced weighted FTS models to solve weighing and recurrence in fuzzy relationship. Lee et al used a two-factor high order FTS. Yu and Lee's methods were applied to predict Taiwan Index (TAIEX).

3. Fuzzy Time Series Models

Let X_t be the time series under study measured at time points $t = 1, 2, 3, \dots, N$. As per the random walk (RW) model widely used to represent financial time series,

$$X_{t+1} = X_t + \varepsilon_t \quad \dots (1)$$

where ε_t is white noise independent of X_t and identically normally distributed with zero mean and variance σ^2 . The parameter σ is constant in homoscedastic models. However the change in variance is highly significant in financial time series. This is due to the fact that periods of uncertainty and relative calm occur in unexpected ways owing to economic, political, psychological events. Several methods have been proposed to handle heteroscedasticity, the change of variance over time. Among these methods, the GARCH model based methods have been widely studied. The RW model gives estimates for probabilistic bounds of future values through Monte Carlo type simulations which are applied in risk management strategies in investing. In case of forecasting, linear models such as exponential smoothing and autoregressive integrated moving average (ARIMA) are benchmark techniques used in connection with RW models.

FTS methods divide the universe of discourse $U = [u_1, u_2, \dots, u_n]$ into several fuzzy sets A_i defined as



$$A_i = \frac{f_{A_1}(u_1)}{u_1} + \frac{f_{A_2}(u_2)}{u_2} + \dots + \frac{f_{A_n}(u_n)}{u_n} \quad \dots (2)$$

where $f_{A_i}: U \rightarrow [0,1]$ is the membership function of the fuzzy set A_i that maps each element to a real number in the unit interval representing its degree of belongingness in the set.

A Fuzzy Time Series on real numbers $Y(t)$ is defined as the collection $F(t)$ of fuzzy sets $f_i(t), (i = 1, 2, \dots)$ that are defined using $Y(t)$ as the universe of discourse.

A Fuzzy Relation between $F(t)$ and $F(t - 1)$ is denoted by $R(t - 1, t)$ and written as

$$F(t) = F(t - 1) \odot R(t - 1, t) \quad \dots (3)$$

If such a relation exist then $F(t)$ is said to be caused by $F(t - 1)$. The variable t denotes time. In short a fuzzy relation is expressed as $F(t - 1) \rightarrow F(t)$. This allows the expression of rules involving linguistic quantities. This enables FTS models to capture human like intelligence. The right hand side is the fuzzy forecast and the left hand side can involve more than one fuzzy sets. If there are N fuzzy sets in the left hand side, it is referred to as a N - order relation. High order relations were introduced by Chen. A group of fuzzy relations is a fuzzy relationship group (FRG).

4. Stock Index Forecasting

Several methods based on neural networks and neuro fuzzy solutions have been proposed in the literature. The mean squared error of forecasting is usually reported. However in order to be useful as a tool for investing, the error must be smaller than naive forecasting of zero returns i.e., the next day's closing price is predicted to be equal to the closing price of the previous day. Table 1 gives the Root mean squared error of forecasting both the price (P) and daily percentage returns (R) of five well traded indices namely Standard & Poor's 500 (S&P 500), Dow Jones Industrial Average (DJIA), National Association of Securities Dealers Automated Quotations (NASDAQ), National Stock Exchange (NSE) of India's National Fifty (NIFTY) and BANKNIFTY. The result is over a 5 year period from 2011-2015. The Non dimensional Error Index (NDEI) is the RMSE divided by standard deviation of the data. It gives a properly scaled version of the RMSE.

Table 1: RMSE of Naive Forecasting

Index	RMSE (P)	RMSE (R)	NDEI (P)	NDEI (R)
S&P500	16.67	0.0269	0.0433	1.4876
DJIA	141.40	0.0182	0.0476	1.4881
NASDAQ	38.14	0.0207	0.0349	1.4667
NIFTY	75.18	0.0209	0.0494	1.3729
BANKNIFTY	188.70	0.0269	0.0456	1.3272

The NDEI values clearly indicate that a good amount of information about a day's closing price is contained in the previous day's closing price and it is much more difficult to predict daily returns from the previous day's returns. The autocorrelation and partial autocorrelation plots of P and R are shown in figure 1 for the case of NIFTY. The autocorrelation plot of prices exhibit strong correlation with every lag. This is due to the fact that the daily change in price is much smaller than the absolute value of the index. Changes in indices occur slowly over a longer period of time than a day. The autocorrelation plot of returns however shows very little correlation with any lag. It is much more difficult to predict daily returns with accuracy which is useful for investing. The partial correlation plots also showcase the same idea.

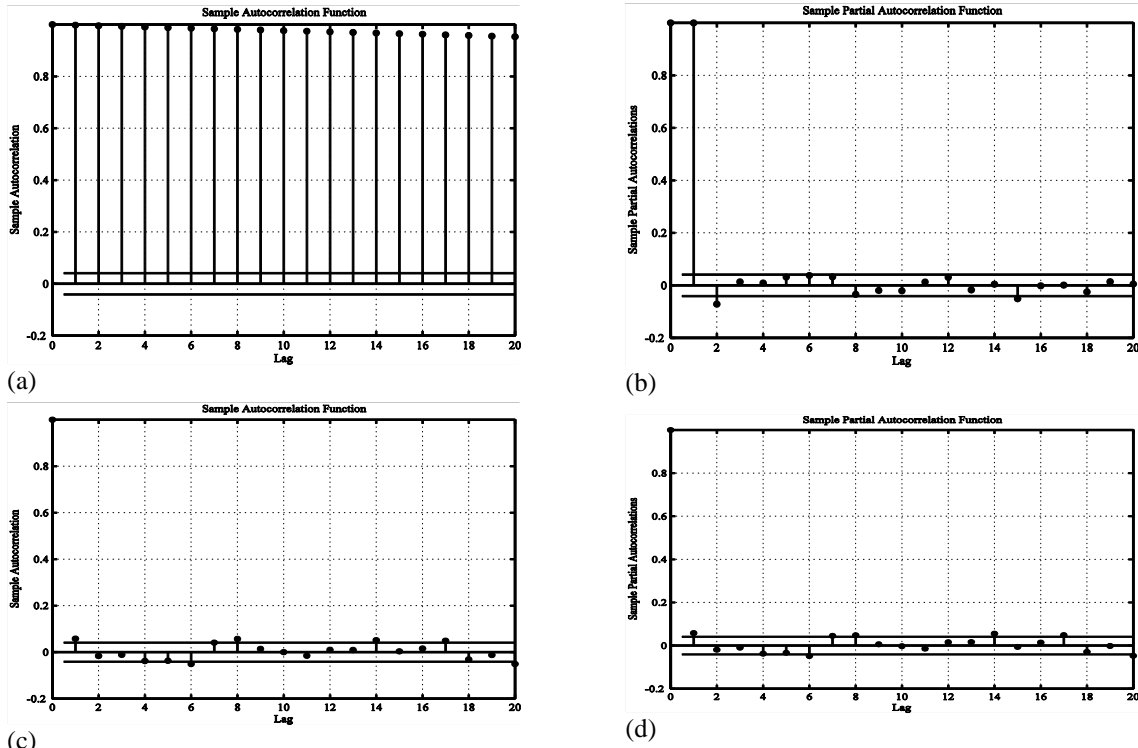


Figure 1. Autocorrelation of (a) Indices (b) Returns and Partial Correlation of (c) Indices and (d) Returns

Using neuro fuzzy techniques lead to overfitting and produce higher errors than the naive method in test data different from training data.

The objective of this paper is to propose a fuzzy regression method which avoids overfitting and produces more accurate results than the naive method. The proposed method is also compared against Adaptive neuro fuzzy inference system (ANFIS). The next section briefly presents the proposed method.

5. Proposed Methodology

Let the daily closing price or index be $P_t, t = 1, 2, 3, \dots, N$.

It is highly correlated with the previous day's closing price which is the naive predictor. It can be written as

$$P_t = \beta_1 P_{t-1} + R_{t-1} \quad \dots (4)$$

The residual R_{t-1} is postulated as a function of the previous day's change in the general mood of the market at the time. The mood is defined by the previous d days price variations. The percentiles of previous d days' prices are used to represent the prevailing mood of the market. In order to capture the support and resistance zones, 5th percentile and 95th percentiles are used. The median or the 50th percentile is also used. The following features are used as independent variables i.e., predictors.

$$X_{1,t-1} = \frac{P_{t-1} - P_{50,t-1d}}{P_{50,t-1d}} \quad \dots (5)$$

$$X_{2,t-1} = \frac{P_{95,t-1d} - P_{t-1}}{P_{50,t-1d}} \quad \dots (6)$$

$$X_{3,t-1} = \frac{P_{t-1} - P_{50,t-1d}}{P_{50,t-1d}} \quad \dots (7)$$

$$X_{4,t-1} = P_{t-1} \quad \dots (8)$$



where

$$P_{t,d} = \text{percentile}_c(P_i, i = t - d, t - d + 1, \dots, t) \quad \dots(9)$$

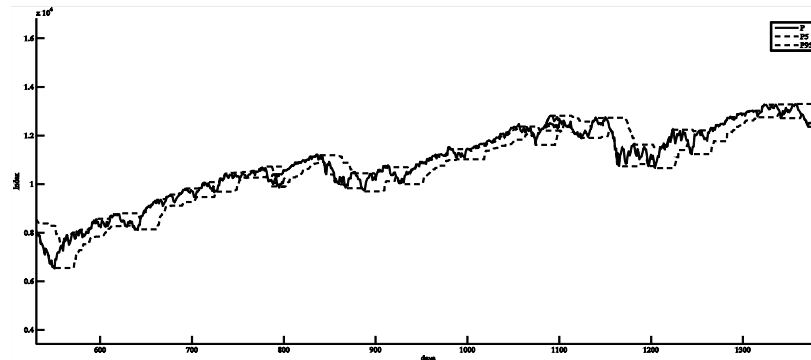


Figure 2: DJIA Series showing P,P₅,P₉₅

The features are shown in figure 2. The idea is that when the price movements are dependent on how close or away it is from the median price of the prevailing mood and whether it is above the resistance or below the support. The idea is illustrated in figure 3.

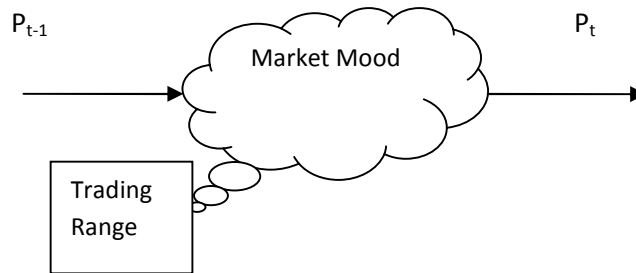


Figure 3: Feature Model of the Proposed Method

A second order linear regression model on P can be written as a

$$P_t = \beta_1 X_{1,t-1} + \beta_2 X_{2,t-1} + \beta_3 X_{3,t-1} + \beta_4 X_{4,t-1} + \varepsilon_t \quad \dots (10)$$

The error term ε_t is assumed to be normally distributed with zero mean. Volatility prediction methods forecasts the variance of ε_t in order to find the bounds of forecast. These bounds are useful in risk management but not in direct investing. The task in this study is to forecast the daily returns itself. If the price fluctuations are adequately captured in the four features then the variance of ε_t can be assumed to be constant i.e., homoscedasticity.

The features are fuzzified into f different fuzzy sets and the fuzzy intervals are determined with the percentiles of values in the training set. Each of the feature is classified into f different fuzzy set with parameters given by $\{[p_1, p_{1+1}, p_{1+2}, p_{1+3}]\}$ where p_i are f equally spaced percentiles on the range of values of each feature.

Trapezoidal membership functions are used. The membership function of a fuzzy set in the range $A = [a, b, c, d]$ is defined as



$$\mu_A(x) = \begin{cases} 0, & (x < a) \text{ or } (x > d) \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \end{cases} \quad \dots (11)$$

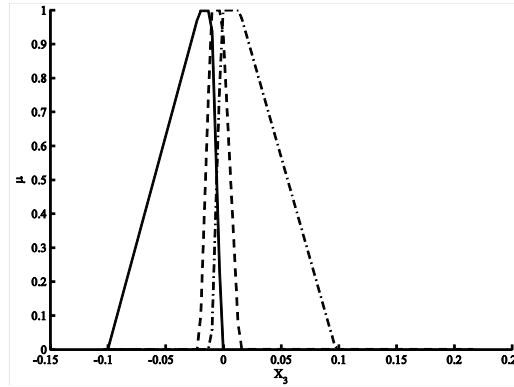


Figure 4: Trapezoidal Membership functions (f=3) of X_3

A set of simplifications is done on standard FTS models to avoid the over fitting problem. A piecewise planar regression is preferred to nonlinear models. The linear regression coefficients are assumed to be different for each of the five fuzzy sets. The least squares solution methodology is modified by weighting the sum of squared errors (SSE) with the membership function for each of the regression. The membership function of pair of values is approximated by the geometric mean of their individual membership functions.

$$\mu_{i,t} = \mu_i(x_{1,t}, x_{2,t}, x_{3,t}, x_{4,t}) = (\mu_i(x_{1,t})\mu_i(x_{2,t})\mu_i(x_{3,t})\mu_i(x_{4,t}))^{\frac{1}{4}} \quad \dots (12)$$

Thus the SSE for the fuzzy set A_i is given by

$$SSE_i = \sum_{t=1}^{N-1} \mu_{i,t-1} (x_t - \beta_{i,1}x_{1,t-1} - \beta_{i,2}x_{2,t-1} - \beta_{i,3}x_{3,t-1} - \beta_{i,4}x_{4,t-1})^2 \quad \dots (8)$$

Separate planar regressions for each of the different moves capture the price dynamics and psychological mood of the markets in the local time period.

The coefficients in equation 6 are obtained by minimizing each of the f regressions as in equation 8.

The least squares solution is obtained by solving the following linear system of equations.

$$M_i x_i = b_i \quad \dots (13)$$

where the matrix M_i , vectors x_i and b_i for the fuzzy set A_i and at time instant t are defined as

$$M_i = \begin{pmatrix} \sum \mu_i x_1^2 & \sum \mu_i x_1 x_2 & \sum \mu_i x_1 x_3 & \sum \mu_i x_1 x_4 \\ \sum \mu_i x_1 x_2 & \sum \mu_i x_2^2 & \sum \mu_i x_2 x_3 & \sum \mu_i x_2 x_4 \\ \sum \mu_i x_1 x_3 & \sum \mu_i x_2 x_3 & \sum \mu_i x_3^2 & \sum \mu_i x_3 x_4 \\ \sum \mu_i x_1 x_4 & \sum \mu_i x_2 x_4 & \sum \mu_i x_3 x_4 & \sum \mu_i x_4^2 \end{pmatrix} \quad \dots (14)$$

$$x_i = \begin{pmatrix} \hat{\beta}_{1,i} \\ \hat{\beta}_{2,i} \\ \hat{\beta}_{3,i} \\ \hat{\beta}_{4,i} \end{pmatrix} \quad \dots (15)$$



$$b_i = \begin{pmatrix} \sum \mu_i P_t x_1 \\ \sum \mu_i P_t x_2 \\ \sum \mu_i P_t x_3 \\ \sum \mu_i P_t x_4 \end{pmatrix} \quad \dots (16)$$

The prediction of P_t is done by regressing on all the different planar regressions of the fuzzy sets. The results are weighted with the membership function value of the predictors and added to get the crisp result.

$$\hat{P}_t = \frac{\sum_{i=1}^4 \mu_{i,t-1} [\beta_{i1} x_{1,t-1} + \beta_{i2} x_{2,t-1} + \beta_{i3} x_{3,t-1} + \beta_{i4} x_{4,t-1}]}{\sum_{i=1}^4 \mu_{i,t-1}} \quad \dots (17)$$

6. Experimental Results and Analysis

6.1 Experimental Setup

The proposed method was implemented in MATLAB and tested on a dataset consisting of historical closing prices of five indices namely S&P 500, DJIA, NASDAQ, NIFTY and BANKNIFTY. The quotes are spot values of the indices from Jan 2007 to Aug 2016. Each of the time series are divided equally into a five sets of two years each for fivefold cross validation. The forecasting accuracy is evaluated through the following performance metrics.

The predicted values of a time series X_t is denoted by \hat{X}_t . The Mean Squared Error (MSE) of the prediction is defined as

$$MSE = \frac{\sum (X_t - \hat{X}_t)^2}{n} \quad \dots (18)$$

Where n is the number of predictions. The Mean Absolute Percent Error (MAPE) of the prediction is defined as

$$MAPE = \frac{100}{n} \sum \left| \frac{X_t - \hat{X}_t}{X_t} \right| \quad \dots (19)$$

The Mean Absolute Deviation (MAD) of the prediction is defined as

$$MAD = \frac{|X_t - \hat{X}_t|}{n} \quad \dots (20)$$

The Largest Absolute Deviation (LAD) of the prediction is defined as

$$LAD = \max |X_t - \hat{X}_t| \quad \dots (21)$$

MSE gives larger weight to large deviations compared to MAD and is desirable in the present scenario. LAD gives the maximum possible deviation and serves as an assurance of maximum possible loss to the investor following the predictor. MAPE gives the error in percentage terms and therefore enables comparison across prediction in different indices. In this work all these four performance measures are reported but MSE, MAPE and LAD are given more attention.

The datasets are displayed in figure 4. The indices dipped lower in 2007-2008 and experienced a bull market in 2008-2013 leading on to range bound volatility on and after 2014. The present work focuses solely on predicting the daily closing price from the previous prices. It is therefore independent of the larger trend in the markets. The forecasting results are discussed in the next section.

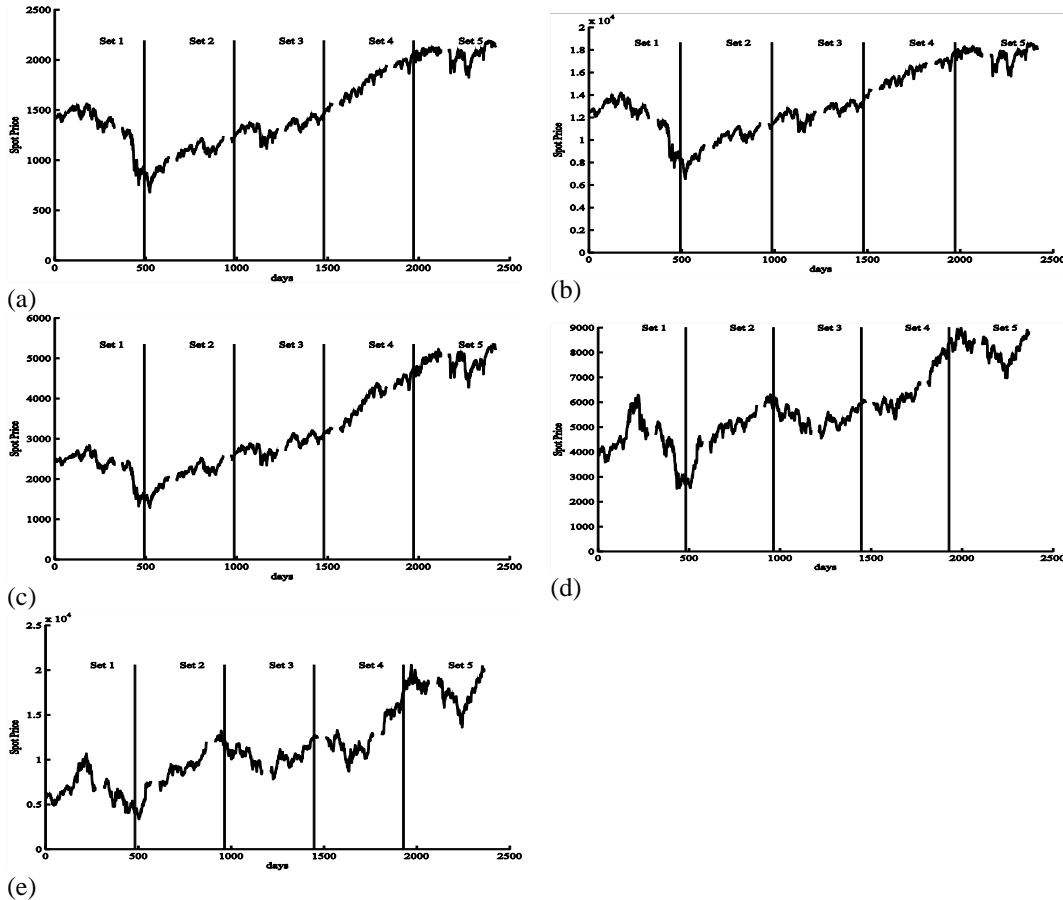


Figure 5: The Datasets (a) S & P 500 (b) DJIA (c) NASDAQ (d) NIFTY 50 (e) BANKNIFTY

6.2 Forecasting Results

Tables 2-5 show the forecasting performance of the proposed method compared against the Naive forecast and ANFIS forecast. The Naive forecast predicts zero change in the spot price of the index every day. Due to the high correlation of the prices, Naive forecast yields good performance. But it cannot be used to benefit from the price changes for investors. Beating the Naive forecast is important for any prediction in order to be useful for investing. ANFIS is a fuzzy forecast method which learns a set of fuzzy rules based on a training set. The data is divided into five equal sets. The methods are trained on one set and tested on another for a total of 20 different combinations for each index. The average performance measures are reported.

Table 2: Forecasting Results on S&P 500

Method	MSE	MAPE	MAD	LAD
NAIVE	226.96	0.6091	10.85	77.68
ANFIS	236.31	0.6216	11.07	79.27
Proposed Method	213.93	0.5914	10.53	75.42

Table 3: Forecasting Results on DJIA

Method	MSE	MAPE	MAD	LAD
NAIVE	16124	0.5855	92.38	619.07
ANFIS	17866	0.6164	97.24	651.65
Proposed Method	15198	0.5685	89.69	601.04

Table 4: Forecasting Results on NASDAQ

Method	MSE	MAPE	MAD	LAD
NAIVE	1605.9	0.7133	28.98	202.06
ANFIS	1779.4	0.7509	30.50	212.69
Proposed Method	1528.5	0.6959	28.27	197.13



Table 5: Forecasting Results on NIFTY 50

Method	MSE	MAPE	MAD	LAD
NAIVE	4570.1	0.7451	50.14	490.95
ANFIS	4681.8	0.7540	50.75	496.91
Proposed Method	4349.9	0.7269	48.92	478.98

Table 6: Forecasting Results on BANKNIFTY

Method	MSE	MAPE	MAD	LAD
NAIVE	41267	1.1059	148.97	1245.4
ANFIS	42275	1.1194	150.78	1260.5
Proposed Method	39049	1.0758	144.92	1211.5

The parameter α is fixed at 10. Accordingly the first α data points are ignored in the calculations. It can be seen that ANFIS does not perform better than naive forecasting. The learned fuzzy rules are over fitted to the training data and perform good results when tested on the same set. However cross validation exposes the poor performance of high order fuzzy systems. The proposed method avoids the over fitting problem by learning different linear rules for different market conditions based on the previous α days data. The improvement in performance is substantial considering the difficulty of the forecasting problem.

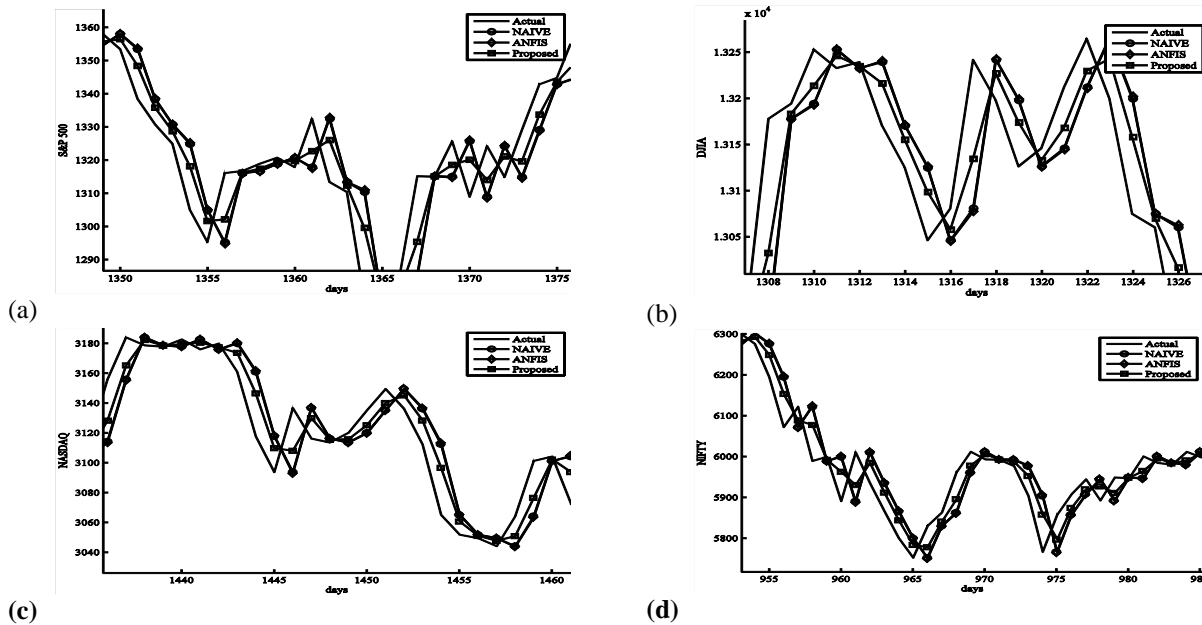


Figure 6: Zoomed in Views of Forecast Values (a) S&P 500 (b) DJIA (c) NASDAQ (d) NIFTY

Figure 5 shows the zoomed in views comparing the forecast of the different methods. All the forecasts trail the actual values because the previous day's price is an important feature. However the proposed method is able to react better and quicker to changes in actual price than Naive and ANFIS methods. A trading strategy based on the proposed method is more likely to yield positive returns than simple trend following techniques. The relative importance of the different features are discussed in the next section.

6.3 Feature Evaluation

The relative importance of the features are analyzed by running the proposed algorithm with different subsets of the four features. The closing price of the previous day $X_{4,t}$ accounts for almost 90% of the predictive power. The other features $X_{1,t}, X_{2,t}, X_{3,t}$ by themselves have no predictive power but when used with $X_{4,t}$ gives a marked improvement in accuracy. The first three features define the market mood and the values of these features serve to separate the different types of market conditions. The membership functions depend on these features thus fitting different regression planes for every unique type of market conditions. Table 7 lists the MAPE values for different subsets of the features.



Table 7: MAPE values for different subsets with the proposed method

Features	S&P 500	DJIA	NASDAQ	NIFTY 50	BANKNIFTY
$X_{1,t-1}$	45.33	42.47	76.92	43.86	71.14
$X_{2,t-1}$	39.51	39.57	72.67	40.42	64.79
$X_{3,t-1}$	44.21	42.52	77.67	44.02	71.21
$X_{4,t-1}$	0.61	0.59	0.71	0.74	1.11
$\{X_{1,t}, X_{2,t}, X_{3,t}\}$	35.14	38.81	70.59	39.78	62.34
ALL	0.5914	0.5685	0.6959	0.7269	1.0758

The accuracy measures of the forecasting methods do not show the effectiveness of the strategy as an investing tool. The profitability can be ascertained through simulation studies. The next section presents a profitability study.

6.4 Profitability Study

This section assesses the performance of the proposed method as an investing strategy. The methods are trained in the first half of the data from 2007 to 2011 and tested on the second half. The Naive Buy and Hold (NBH) strategy involves buying the index at the beginning of the period and selling at the end. The Simple Trend Following strategy involves going long or short at the end of every day depending on whether the index has increased or decreased on the day. The predictions of ANFIS based method and the proposed method are used to either go long or short at the close of every day. The various statistics of the returns are presented in table 8.

The results in table 8 illustrates a number of interesting facts about stock index prediction. It is difficult to match the net profit or loss of the NBH strategy. This is true especially in the well developed and competitive indices of S&P 500, DJIA and NASDAQ. However the disadvantage of the NBH is that the capital is locked for a unknown period of time. If the longs were made before a major downturn, the investors have to wait for several years before they can liquidate the accounts. Daily longs and shorts enable close watching and controlled evolution of the accounts. The simple trend following strategy captures all the large movements that occur together for more than a day. This means that the black swan movements [31] that define the evolution of the indices are not missed. However range bound markets are known for alternating movements called whipsaws which can reduce the returns of trend following strategy at the daily level. The proposed method outperforms simple trend following and ANFIS predictions. In case of NIFTY and BANKNIFTY it outperforms NBH also. In case of NIFTY the net returns are three times and in case of BANKNIFTY two times that of NBH. All the strategies except NBH assume that the investors close their positions and initiate fresh positions at the close of every working day. The maximum drawdown of the accounts are also shown.

Table 8: Profitability Results of the compared strategies in index points

Index	Return Stats	Buy and Hold	Simple Trend Following	ANFIS	Proposed Method
S & P 500	No. of Trades	1	1230	1230	1230
	% Wins	100.00	47.56	47.48	49.76
	Total Profits	944.94	6382.10	6360.40	6771.40
	Total Losses	0.00	6953.90	6975.60	6564.60
	Net P/L	944.94	-571.80	-615.20	206.71
	Max Drawdown	0.00	-571.80	-615.20	-75.60
DJIA	No. of Trades	1	1228	1230	1230
	% Wins	100.00	47.07	46.91	48.70
	Total Profits	6644.70	54141.00	53875.00	57835.00
	Total Losses	0.00	59146.00	59645.00	55686.00
	Net P/L	6644.70	-5005.00	-5770.00	2149.00
	Max Drawdown	0.00	-5005.00	-5770.00	-653.74
NASDAQ	No. of Trades	1	1230	1230	1230
	% Wins	100.00	49.19	48.94	50.57
	Total Profits	2692.90	17319.00	17075.00	18056.00
	Total Losses	0.00	18317.00	18561.00	17581.00
	Net P/L	2692.90	-998.00	-1486.00	475.00
	Max Drawdown	0.00	-1031.00	-1518.60	-346.78
NIFTY	No. of Trades	1	1202	1203	1203
	% Wins	100.00	53.99	53.78	56.03



	Total Profits	3801.10	33508.00	33268.00	35535.00
	Total Losses	0.00	26768.00	27023.00	24756.00
	Net P/L	3801.10	6740.00	6245.00	10779.00
	Max Drawdown	0.00	-491.50	-491.50	-294.30
BANKNIFTY	No. of Trades	1	1203	1203	1203
	% Wins	100.00	52.78	52.78	54.03
	Total Profits	10840.00	100220.00	100220.00	104320.00
	Total Losses	0.00	79141.00	79141.00	75043.00
	Net P/L	10840.00	21079.00	21079.00	29277.00
	Max Drawdown	0.00	-86.40	-86.40	-86.40

6.5 Computational Efficiency

The proposed method requires the solution of four simultaneous equations which take constant time. The summations required to form the set of linear equations is linear in the number of training data points i.e., the proposed method runs in $O(n)$ time. The complex calculations of min-max operators in fuzzy time series models are avoided and a simpler trapezoidal membership functions are used with fixed percentiles as break points. The algorithm required less than a second to execute on a Intel iCore5 system with 4 GB RAM to train on 500 data points.

7. Conclusion and Future Work

This paper presented a simple piecewise planar regression solution to forecast stock indices. The daily closing price of five well traded indices was used. The closing price of the previous day is highly correlated and accounts for most of the prediction accuracy. However the naive method of using the previous time series point as a predictor is not useful from the investors' standpoint. The proposed method was compared with naive and ANFIS methods. It was shown that the proposed method gave better results than the other methods. The profitability studies indicated the effectiveness of the proposed method as a tool of investing. Future research work may be directed in evaluating better trading strategies based on the proposed method. Also forecasting accuracy may be improved by identifying different market periods with input from fundamental market analysis.

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