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EFFECT OF HEAT TRANSFER PAST A STRETCHING TEMPERATURE DEPENDENT VISCOSITY ON BOUNDARY LAYER SLIP FLOW AND HEAT SOURCE /SINK

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Abstract

The steady boundary layer slip flow and heat source/sink on heat transfer past a stretching sheet are studied when the fluid viscosity varies linearly with temperature. The self- similar equations are obtained using similarity transformations and then those are solved numerically by shooting method. The analysis reveals that with the increasing values of viscosity parameter the viscous boundary layer thickness increases and oppositely, the thermal boundary layer thickness decreases. For the increase of the value of slip parameter the velocity decreases up to a crossing over point and increases after that point. The temperature increases when the value of slip parameter increases. The magnitude of the skin- friction coefficient and the rate of heat transfer decrease with the increasing values of slip parameter.

Keywords: Boundary Layer Flow, Partial Slip, Heat Transfer, Stretching Sheet, Heat Source/Sink, Temperature Dependent Viscosity.

Introduction

The study boundary layer flow of an incompressible viscous fluid over a stretching sheet has many applications manufacturing industries and technological processes. Such as, glass fiber production. Wire drawing, paper production, metal and polymer processing industries and many others. Crane (1970) first considered the steady laminar boundary layer flow of a Newtonian fluid caused by a linearly stretching flat sheet and found an exact similarity solution in closed analytical form. Wang (1984) investigated the three- dimensional flow due to the stretching surface.

In all the above mentioned flow problems, the thermo physical properties of fluid were assumed to be constant. However, it is noticed that these properties. Especially the fluid viscosity may change with temperature. In order to appropriately model the flow and heat transfer phenomena, it becomes essential to consider the variation of fluid viscosity due to temperature. Lai and Kulacki[1] considered the effects of variable viscosity on convective heat transfer along a vertical surface in porous medium. Pop et al.[2] discussed the influence of variable viscosity on laminar boundary layer flow and heat transfer due to a continuously moving flat plate. Abel et al.[3] investigated the flow of visco-elastic fluid and heat transfer over a stretching sheet with variable viscosity. Mukhopadhyay et al.[4] analyzed the effects of temperature dependent viscosity on MHD boundary layer flow and heat transfer over stretching sheet. EI-Aziz[5] studied the flow, heat and mass transfer characteristics of a viscous electrically conducting fluid having temperature dependent viscosity and thermal conductivity past a continuously stretching surface, taking into account of the effect of Ohmic heating. Further, some very important investigations regarding the variable viscosity effects on the flow and heat transfer over stretching sheet under different physical condition were made by Pantokratoras[6] , Mukhopadhyay and Layek[7,8] and Mukhopadhyay[9].

The non- adherence of the fluid to a solid boundary, also known as velocity slip, is phenomenon that has been observed under certain circumstances (Yoshimura and Prudhomme [10]. It is a well-known fact that, a viscous fluid normally sticks to the boundary. But, there are many fluids, e.g. particulate fluids, rarefied gas etc., where there may be slip between the fluid and the boundary (Shidlovskiy [11]. Beavers and Joseph [12] proposed a slip flow condition at the boundary. Of late, there has been a revival of interest in the flow problems with partial slip (Andersson [13]; Ariel [14].Wang [15] undertook the study of the flow of a Newtonian fluid past a stretching sheet with partial slip and purportedly gave an exact solution. Bhattacharyya et al.[16]showed the slip effects on the dual solutions of stagnation-point flow and heat transfer towards a stretching sheet. The unsteady flow over shrinking sheet with slip boundary condition was investigated by Mukhopadhyay and Andersson [17]. Mukhopadhyay [18] and Bhattacharyya et al.[19].

The purpose of present study, the effects of partial slip on steady boundary layer flow and heat transfer past a stretching sheet with variable viscosity and heat source /sink are investigated. The fluid viscosity is assumed to vary linearly with the temperature. The governing partial differential equations by the application of similarity transformations. Then, the transformed equations are solved numerically by shooting , method. The results are presented in the form of figures and discussed physically in all contexts.

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Mathematical Formulation

Consider a steady, two dimensional slip flow of a viscous incompressible fluid past a linearly stretching sheet with temperature dependent viscosity. Using boundary layer approximation, the equations of motion and the energy equation may be written in usual notation as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Linear Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) \qquad (3)$$

where u and v are velocity components in x - and y – directions respectively,ρ is the fluid density. μ is the coefficient of fluid viscosity, v ($=\frac{\mu}{\rho}$) is the kinematic fluid viscosity,T_w is the temperature sheet , T is the temperature. K is the fluid thermal conductivity and C_p is the specific heat.T_∞ is the free stream temperature, Q_0 is the volumetric rate of heat generation or absorption.

The appropriate boundary conditions for the velocity components and the temperature are given by

$$U = cx + L\left(\frac{\partial u}{\partial y}\right),\ v = 0 \text{ at } y = 0;\ u \to \infty \qquad (4)$$

$$\text{And } T = T_w \text{ at } y = 0;\ T = T_\infty \text{ as } y \to \infty, \qquad (5)$$

Now the stream function ψ (x ,y) is introduced as:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \qquad (6)$$

The temperature dependent viscosity of the fluid is of the form [20, 22]

$$\mu = \mu^*[a+b(T_w - T)] \qquad (7)$$

Where $\mu^*$is the constant value of the coefficient of viscosity in the free stream and a,b are constants with b(>0)having unit K^{-1}.

Here, we apply the viscosity temperature relation $\mu = a^*-b^*T$ which accords with the relation $\mu = e^{-a^*T}$[36] when second and higher order terms are neglected form the expansion.

The expression of kinematic viscosity becomes $v = v^*[a+b(T_w - T)]$. where $v^* = \mu^*/\rho$ is the constant value of the kinematic fluid viscosity.

Now for Relation (6), the continuity Equation (1) is satisfied automatically. Using (6) and (7). The momentum Equation (2) and the temperature Equation (3) take the following forms:

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = -v^*b\frac{\partial T}{\partial y}\frac{\partial^2 \psi}{\partial y^2} + v^*[a+b(T_w - T)]\frac{\partial^3 \psi}{\partial y^3} \qquad (8)$$

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And

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p}(T - T_\infty) \quad (9)$$

The boundary conditions in (4) of the flow reduce to

$$\frac{\partial\psi}{\partial y} = cx + L\frac{\partial^2\psi}{\partial y^2},\ \frac{\partial\psi}{\partial x} = 0 \text{ at } y = 0;\ \frac{\partial\psi}{\partial y} \to 0 \text{ as } y \to \infty \quad (10)$$

Next, we introduce the dimensionless variables for ψ and T as:

$$\Psi = \sqrt{cv^*}\, x f(\eta) \text{ and } T = T_\infty + (T_W - T_\infty)\,\theta(\eta). \quad (11)$$

$$Pr = \frac{\mu c_p}{\kappa},\ Q = \frac{Q_0}{\rho c_p c}$$

Where the similarity variable η is defined as $\eta = y\,(c/v^*)^{1/2}$, f(η) is the dimensionless stream function ,θ the dimensionless temperature, η – the similarity variable, Pr – the Prandtl number, Q – the heat source (Q >0 or sink (Q<0) parameter.

Using (11) we finally obtain from (8) and (9) following self – similar equations as:

$$(a+A-A\theta)f''' + ff'' - A\theta' f'' - f'^2 = 0 \quad (12)$$

$$\text{and } \theta'' + Pr\, f\, \theta' + Pr\, Q\theta = 0 \quad (13)$$

Where $A = b(T_W - T_\infty)$ is the viscosity parameter .

The boundary conditions in (10) and (5) reduce to the following forms:

$$f(\eta) = 0,\ f'(\eta) = 1+\delta f''(\eta) \text{ at } \eta=0;\ f'(\eta) \to 0 \text{ as } \eta \to \infty \quad (14)$$

And

$$\theta(\eta) = 1 \text{ at } \eta = 0;\ \theta(\eta) \to 0 \text{ as } \eta \to \infty \quad (15)$$

Where $\delta = L\,(c/v^*)^{1/2}$ is the slip parameter.

Numerical Method for Solution

The nonlinear coupled differential Equations (12 and (13) along with the boundary Conditions (14) and (15) form a two point boundary value problem (BVP) and are solved using shooting method [5,6,7,8,9] by converting it into an initial value problem (IVP). In this method. It is necessary to choose a suitable finite value of $\eta \to \infty$. Say η_∞.We set following first- order system:

$$f' = p,\ p' = q,\ q' = (p^2 - fq + Azq)/(a+A - A\theta) \quad (16)$$

$$\text{and } \theta' = z,\ z' = -Pr\, fz,\ z'' = -PrfQ \quad (17)$$

With the boundary conditions

$$f(0) = 0,\ p(0) = 1 + \delta q(0),\ \theta(0) = 1 \quad (18)$$

To solve (16) and (17) with (18) as an IVP we must need vlues for q(0) i.e. f''(0) and z(0) i.e.θ'(0) but no such values are given . The initial guess values for f''(0) and θ'(0) are chosen and applying fourth order Runge – Kutta method a solution is obtained .We compare the calculated values of f'(η) and θ(η) at η_∞(=20, in all cases) with the given boundary conditions $f'(\eta_\infty)=0$ and $\theta(\eta_\infty)=0$ and adjust values of f'(0) and θ(0) using "secant method" to give better approximation for the solution.

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Results and Discussion

To analyze the results, the numerical computations have been performed for several values of the physical parameters, namely, the viscosity parameter A, the slip parameter δ and the Pranstl number Pr. The value of a is kept constant (=1) in whole analysis. The computed values are displayed in the form of figures and the physical clarifications are rendered for every case.

FIG.1 Since the viscosity is temperature dependent, the Prandtl number Pr also affects the velocity fields in addition with the temperature distribution and this can be observed from Fig.1 and Fig.2.Fig.1 reveals that the velocity at a particular η increases with increase in Pr. Also, the temperature at a point and the thermal boundary layer thickness decrease when Pr increases [Fig.1]. The effects of Prandtl number, especially, on the velocity field are very important and realistic in flow dynamics.

Fig.3 illustrates the effect of the heat source of sink parameter (Q) on the temperature. It is noticed that as the heat source or sink parameter increases, the temperature increases and the lower value of Q=-2, high temperature reached.

η

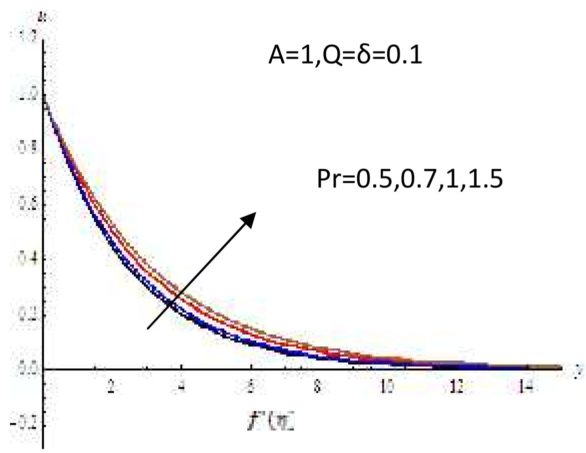
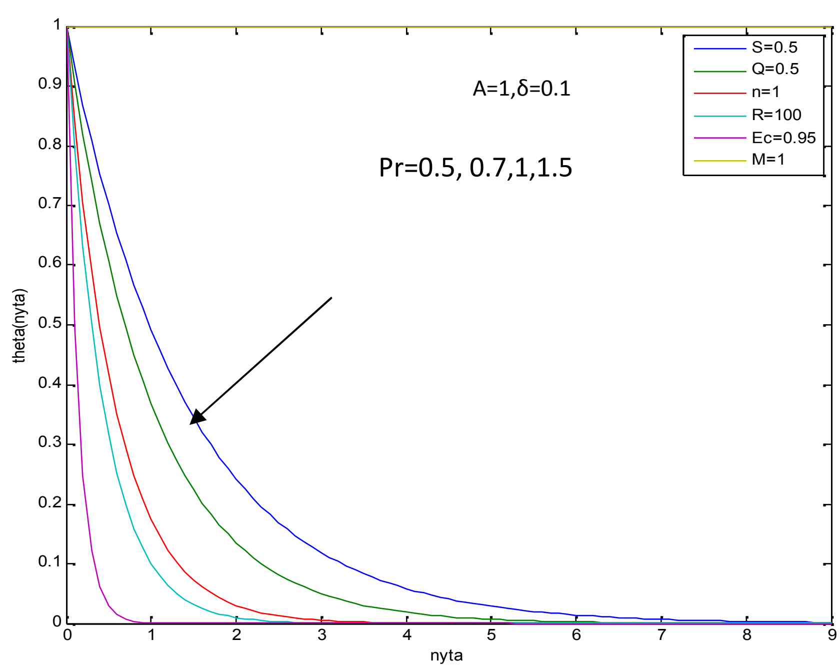


Fig.7 Velocity Profiles For Various Values of Pr

θ(η)



η

Fig.8 Temperature Profiles For Various Values of Pr

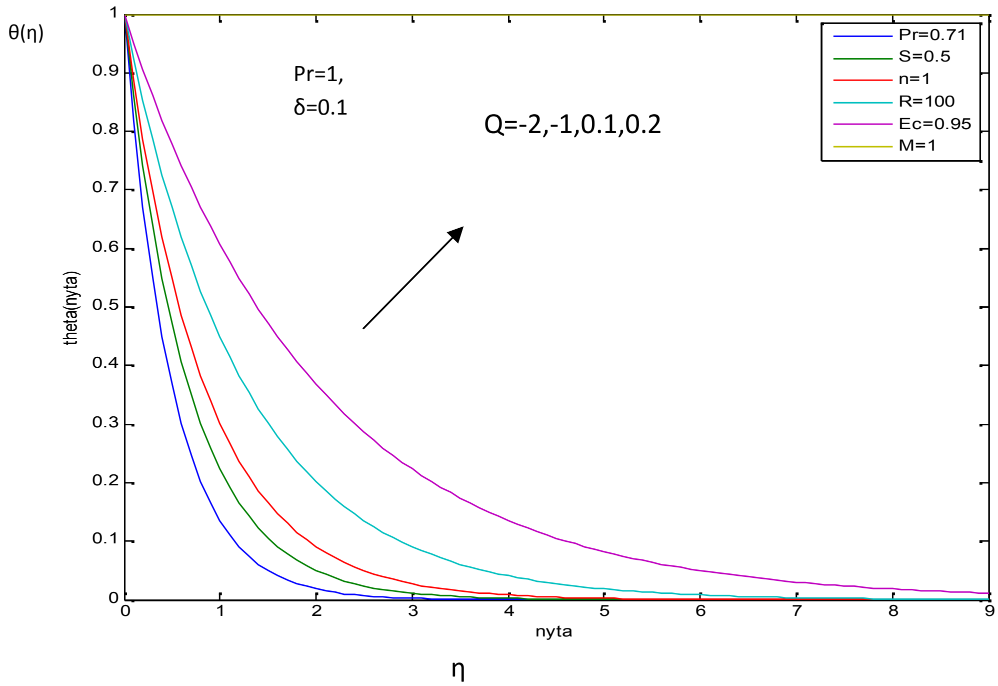


Fig.3 Temperature for Different Values of Q

Conclusions

In order to get physical inside into the problem, we have calculated the velocity field, temperature field,prandtl number(Pr), is the ratio of viscous forces to assigning specific values to the different values to the parameters involved in the problem, as the Prandtl number increases, and heat source sink(Q) parameter decreases.

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