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DISPERSION OF A POORLY CONDUCTING COUPLE STRESS FLUID BETWEEN TWO IMPERMEABLE RIGID PARALLEL PLATES USING GENERALIZED DISPERSION MODEL

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Abstract

In this paper a mathematical model is presented which describes the dispersion of a poorly conducting couple stress fluid in laminar flow between two impermeable rigid parallel plates embedded with segmented electrodes. By using generalized dispersion model, the validity of time-dependent dispersion co-efficient is widened. The effect of couple stress parameter 'a' and electric number 'We' on the most dominant dispersion coefficient is studied. The exact solution for the dimensionless mean concentration distribution is obtained as a function of dimensionless time, axial distance and couple stress parameter. The results of pure convection are also reported. It is shown that the effect of couple stress parameter is to decrease the magnitude of dispersion coefficient thereby increasing the concentration distribution whereas increase in electric number, increases the dispersion coefficient thereby decreasing the concentration distribution.

Keywords: Generalized Dispersion, Couple Stress, and Poorly Conducting Fluid.

1. Introduction

The mathematical models involving dispersion phenomena are necessary and indispensable tools to facilitate the decision making process for citing of industrial and residual complexes and for the purpose of emergency response management and impact assessment studies. On a regular basis, especially in developing countries like India it is not feasible to operate the conventional and expansive observational network to observe the concentration of pollutants originating from various vulnerable locations particularly in biomedical engineering process. Dispersion of pollutants has been affected by physical, chemical and mechanical nature of waste materials, the location of the pile of waste and the nature of terrain downwards from the stack. Using Fick's law, the concentration distribution of pollutants due to above waste material is obtained in the form of concentration equation in the mathematical model of dispersion. To study the dispersion, various models are identified as Taylor's (1953) dispersion model valid for large time, Airs(1956) dispersion model which is improvement over Taylor's dispersion model, Generalized plume/puff model etc. However these models have inherent limitations like applicable to complex terrain, accountability of realistic flow, low and variable flow conditions. Further the solution in the analytical models can be obtained only for the simplest form of flow conditions. Therefore in this paper we use generalized dispersion model which is so general valid for all time that we can obtain the results of other dispersion models.

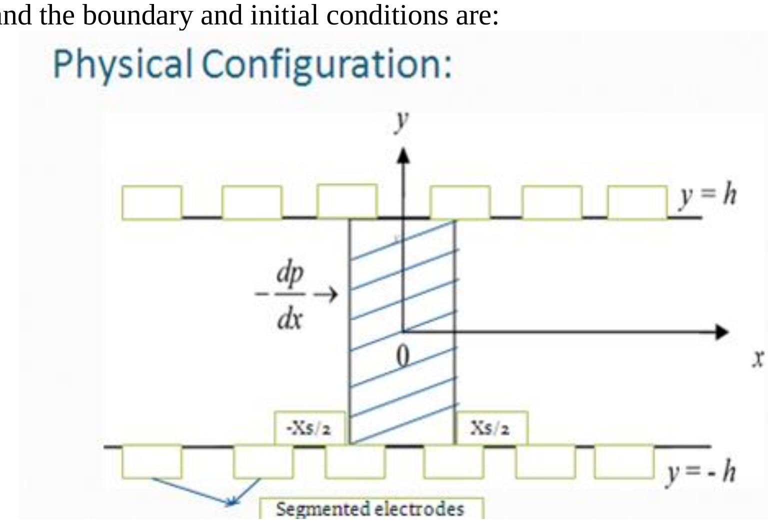
We study an unsteady dispersion in a couple stress poorly conducting fluid in the presence of a transverse electric field with the motivation of understanding the haemolysis caused by artificial organs made of metals in biomedical engineering. Poorly conducting fluid means fluid whose electrical conductivity is a strong function of temperature, concentration and combination of both temperature and concentration and increases with temperature with electrical conductivity $\sigma << 1$. The difference in temperature produces difference in conductivity. This difference in conductivity releases the free charges, resulting in induced electric field $\vec{E}_i$. In addition to these there may be an applied electric field $\vec{E}_a$ due to embedded electrodes of different electric potentials at the boundaries. The total electric field $\vec{E}$ namely the sum of induced and applied electric fields $\vec{E} = \vec{E}_i + \vec{E}_a$ produce a current which acts as sensing. This total electric field together with distribution of charges produces an electric force $\rho_e\vec{E}$, which acts as actuator. Sensing and actuation are two important properties of smart materials.

The effective functioning of microfluidic devices in electronics, electrical and mechanical engineering involving fluids, particularly those having vibrations and petroleum products containing organic, inorganic, and other micro fluidics, requires the understanding and control of stability of parallel fluid flows. These substances, dissolving in the fluid, make the fluid poorly conducting. The electrical conductivity σ of such poorly conducting fluidics, increases forming micro polar fluid. According to Eringen (1966), the micro polar fluids may be regarded as non-Newtonian fluids. These freely suspended particles in fluid spin, producing micro rotation. Fluids with anti-symmetric stress are known as micro polar fluids. Couple stress fluids are the particular case of micro polar fluids wherein, unlike most micro polar fluids, there is a mismatch between the spin of the suspension and the vorticity of the suspending fluid.

Rudraiah et al (2005, 2006) have shown that self-generated electric field reduces the concentration of RBCs and hence increases dispersion. Siddeshwar et al (1989) have studied the effects of couple stress and magnetic field on unsteady convective diffusion in a rectangular channel. The study of dispersion in a poorly conducting fluid have been done by Rudraiah et al (2014, 2015, 2018) and Rudraiah et al (1986) have studied the effect of couple stress on the dispersion of erythrocytes in a channel bounded by rigid walls and showed that the couple stress augments haemolysis. Therefore, in a study involving the control of haemolysis it is important that the combined effect of couple stress and poorly conducting nature have to be taken into account. The results so obtained are useful in the design of efficient artificial organs free from impurities. Therefore, the objective of this paper is to consider these effects in the study of the unsteady convective diffusion of RBCs in the physiological fluid modeled as poorly conducting couple stress fluid, using the Generalized Dispersion approach of Gill and Sankarasubramanian (1970) and compare our results with the results obtained by Rudraiah et al (2011) where they have studied electro hydrodynamic dispersion of poorly conducting couple stress fluid bounded by porous layers. To achieve this objective the paper is planned as follows. The required basic equations, the boundary and initial conditions are specified in section 2. The generalized dispersion coefficient is determined in section 3. Results and Discussions are reported in section 4. We found that an increase in the couple stress parameter and decrease in electric number, decreases the axial dispersion coefficient and hence preserves the accumulation of RBC thus preventing haemolysis.

2. Mathematical Formulation

We assume the flow of a poorly conducting couple stress fluid to be laminar, fully developed and unidirectional with a uniform axial pressure gradient. In the absence of external constraints, the basic equations following Rudraiah et al (2011) and the boundary and initial conditions are:



Basic Equations for the flow is

$$0=\frac{-\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}-\lambda\frac{\partial^4 u}{\partial y^4}+\rho_e E_x \quad (2.1)$$

u is velocity in the x-direction in the free flow, p the pressure, μ the viscosity of couple stress fluid, λ the couple stress coefficient in the free flow

The mass balance equation in a fully developed flow is

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \quad (2.2)$$

where C is the concentration, D the diffusion co-efficient.

The boundary conditions on the velocity are

u=0 at $y=\pm h$ (2.3.1)

$$\frac{d^2 u}{dy^2}=0 \quad at \quad y=\pm h \quad (2.3.2)$$

The boundary conditions on concentration are

$C(0,x,y)$ = C_0 for $|x|\le(1/2)x_s$ (2.4.1)

$C(0, x, y) = 0$ for $|x|>(1/2)\,x_s$ (2.4.2)

$$\frac{\partial C}{\partial y}(t,x,h)=0 \quad (2.4.3)$$

$$\frac{\partial C}{\partial y}(t,x,-h)=0 \quad (2.4.4)$$

$$C(t,\infty,y)=\frac{\partial C}{\partial y}(t,\infty,y)=0 \quad (2.4.5)$$

$C(t, x, y)$ = finite. (2.4.6)

where C_0 is the concentration of the initial slug input of length x_s and (2.4.1 & 2.4.2) represent the initial concentration, (2.4.3 & 2.4.4) specifies that there is no transfer of mass flux at the walls, and (2.4.5 & 2.4.6) specifies that the concentration does not reach points far away downstream.

We now introduce the following dimensionless variables in to (2.1), (2.2), (2.3) and (2.4)

$$U=\frac{u}{\bar{u}},Y=\frac{y}{h},\ X=\frac{x}{hPe},\ \theta=\frac{C}{C_0},\ \tau=\frac{t\bar{u}}{L}\ \ \ell^2=\frac{\lambda}{\mu}\ a^2=\frac{\mu h^2}{\lambda}\ \ \rho_e^*=\frac{\rho_e}{\left(\frac{\in_0 V}{h^2}\right)},\ E_x^*=\frac{E_x}{\left(\frac{V}{h}\right)} \quad (2.5)$$

where $\bar{u}$ is the average velocity of the flow.

The dimensionless basic velocity equation for the flow introducing $\rho_e E_x=\frac{PeX_0\alpha^2 e^{-\alpha Y}}{(e^{\alpha}-e^{-\alpha})}+\frac{PeX_0\alpha(1-\alpha Y)}{2}(\because \alpha<<1)$ following Rudraiah et al (2011) is given by

$$\frac{d^4 U}{dY^4}-a^2\frac{d^2 U}{dY^2}=-Ka^2(B_1-B_2+B_2\alpha Y) \quad (2.6)$$

The dimensionless concentration equation is given by

$$\frac{\partial\theta}{\partial\tau}+U^*\frac{\partial\theta}{\partial\xi}=\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial\xi^2}-\frac{\partial^2\theta}{\partial\xi^2} \quad (2.7)$$

where $B_1 = \frac{dp}{dX}, B_2 = \frac{WePe^2 \alpha X_0}{2}, l = \sqrt{\frac{\lambda}{\mu}}$, $a = h/l$ is the couple stress parameter, $K = D/\gamma$ is the ratio of mass diffusion to kinematic viscosity, $\gamma = \mu/\rho$, $We = \varepsilon_0 \left(V/h^2\right)/\rho\bar{u}^2$ is the electric number, $\text{Pe} (= \bar{u}\ h/D)$ is the Peclet number, $U^* = \left(U - \bar{U}\right)/\bar{U}$ (non-dimensional velocity in a moving coordinate) and $\xi = \left(X - \tau\right)/L$ is the dimensionless axial coordinate moving with the average velocity $\bar{U}$.

The non-dimensional boundary conditions on velocity and couple conditions are

U = 0 at $Y = \pm 1$ (2.8)

$$\frac{d^2U}{dY^2} = 0 \quad at \quad Y = \pm 1 \tag{2.9}$$

The dimensionless initial and boundary conditions for θ are:

$$\theta(0, X, Y) = 1 \text{ for} |x| \le (1/2) x_s$$
$$\theta(0, X, Y) = 0 \text{ for} |x| > (1/2) x_s$$
$$\frac{\partial \theta}{\partial Y}(\tau, X, 1) = 0 \tag{2.10}$$
$$\frac{\partial \theta}{\partial Y}(\tau, X, -1) = 0$$
$$\theta(\tau, \infty, Y) = \frac{\partial \theta}{\partial Y}(\tau, \infty, Y) = 0$$

3. Generalized Dispersion Coefficient:

The solution of equation (2.1) using (2.3.1) is

$$U = \frac{-K(B_1 - B_2)}{2}\left[1 - Y^2 - \frac{2}{a^2}\left(1 - \frac{\cosh aY}{\cosh a}\right)\right] - \frac{KB_2 \alpha \sinh aY}{a^2 \sinh a} + A_1 + A_2 Y + A_3 Y^2 \tag{3.1}$$

where $A_1 = KB_2\alpha\left\{\frac{1}{a^2} - \frac{co\,\text{th}\, a}{a} + \frac{1}{3}\right\}, A_2 = KB_2\alpha\left\{\frac{co\,\text{th}\, a}{a} - \frac{1}{2}\right\}, A_3 = \frac{KB_2\alpha}{6}$

Here

$$U^* = \frac{1}{2}\frac{\left\{1 - 3Y^2 + \frac{6}{a^2 \cosh a}\left(\cosh aY - \frac{\sinh a}{a}\right) + A_4\left(A_5 \sinh aY - A_2 Y - A_3 Y^3\right)\right\}}{A_6}$$

where $A_4 = \frac{6}{K(B_1 - B_2)}$, $A_5 = \frac{KB_2\alpha}{a^2 \sinh a}$, $A_6 = 1 + \frac{3}{a^2}\left(\frac{\tanh a}{a} - 1\right) - \frac{3A_1}{K(B_1 - B_2)}$

The solution of equation (2.2) is written as a series expansion in the form (Using Gill and Sankarasubramanian 1970)

$$\theta(\tau, \xi, Y) = \theta_m(\tau, \xi) + \sum_{k=1}^{\infty} f_k(\tau, Y)\frac{\partial^k \theta_m}{\partial \xi^k} \tag{3.2}$$

where $\theta_m = \frac{1}{2}\int_{-1}^{1} \theta \, dY$ the dimensionless cross sectional average concentration.

We now assume that the process of distributing θ is diffusive in nature right from time zero (unlike the models of Taylor (1953) Aris (1956) Lighthill (1969)). One can introduce the Generalized dispersion model with time-dependent dispersion coefficient as

$$\frac{\partial \theta_m}{\partial \tau} = \sum_{k=1}^{\infty} K_k \frac{\partial^k \theta_m}{\partial \xi^k} \tag{3.3}$$

Equation (3.3) is solved subject to the conditions

$$\theta_m(0,\xi) = 1 \quad |\xi| \le (1/2) X_s, \tag{3.4.1}$$

$$\theta_m(0,\xi) = 0 \quad |\xi| > (1/2) X_s \tag{3.4.2}$$

$$\theta_m(\tau,\infty) = 0 \tag{3.4.3}$$

The most dominant dispersion coefficient following Gill and Sankarasubramanian (1970) is

$$\begin{aligned}
K_2(\tau) = {} & \frac{1}{Pe^2} + \frac{1}{8A_6^2}\Bigg(\frac{16}{945} - \frac{4}{135}\frac{\tanh a}{a^7}(12a^4 - 180a^2 - 135) - \frac{20}{a^6} + \frac{4}{3a^6}\left(\frac{\tanh a}{a}\right)^2 (5a^2 + 12) \\
& - \sum_{n=1}^{\infty} \frac{16\, e^{-\lambda_n^2 \tau}}{\lambda_n^2}\left(\frac{1}{\lambda_n^2} - \frac{\tanh a}{a(\lambda_n^2 + a^2)}\right)^2 - \frac{8}{3}\sum_{n=1}^{\infty} \frac{e^{-\lambda_n^2 \tau}}{\lambda_n^2}\left(\frac{1}{\lambda_n^2} - \frac{\tanh a}{a(\lambda_n^2 + a^2)}\right) \\
& \left(\frac{-A_4 A_2}{2} - \frac{A_4 A_3}{4} + F_0 + \frac{A_4 A_2 + 3A_4 A_3}{\lambda_n^2} - \frac{6A_4 A_3}{\lambda_n^4} + \frac{\lambda_n^2 A_4 A_5 \cosh a}{a(\lambda_n^2 + a^2)}\right) \\
& - \frac{8}{3}\sum_{n=1}^{\infty} \frac{e^{-\lambda_n^2 \tau} \cos \lambda_n}{\lambda_n^2}\left(\frac{1}{\lambda_n^2} - \frac{\tanh a}{a(\lambda_n^2 + a^2)}\right)\left(\frac{-A_4 A_2}{\lambda_n^2} + \frac{6A_4 A_3}{\lambda_n^4} - F_0 - \frac{\lambda_n^2 A_4 A_5}{a(\lambda_n^2 + a^2)}\right) \\
& + \frac{A_4^2 A_5^2}{18a^2}\left(\frac{\sinh 2a}{a} - 2\right) - \frac{A_4^2 A_5 A_2}{54}\left(\frac{2\cosh a}{a} - \frac{6\sinh a}{a^2} + \frac{12\cosh a}{a^3} - \frac{12\sinh a}{a^4}\right) \\
& - \frac{A_4^2 A_5 A_3}{90}\left(\frac{\cosh a}{a} - \frac{5\sinh a}{a^2} + \frac{20\cosh a}{a^3} - \frac{60\sinh a}{a^4} + \frac{120\cosh a}{a^5} - \frac{120\sinh a}{a^6}\right) \\
& + \frac{2}{9} F_0 A_4 A_5\left(\frac{\cosh a}{a} - \frac{\sinh a}{a^2}\right) - \frac{2}{9}\frac{A_2 A_4^2 A_5}{a^2}\left(\frac{\cosh a}{a} - \frac{\sinh a}{a^2}\right) + \frac{2A_4 A_2^2}{270} + \frac{A_2 A_4^2 A_3}{630} \\
& - \frac{2A_2 A_4 F_0}{27} - \frac{2A_4^2 A_5 A_3}{9a^2}\left(\frac{\cosh a}{a} - \frac{3\sinh a}{a^2} + \frac{6\cosh a}{a^3} - \frac{6\sinh a}{a^4}\right) \\
& + \frac{A_4^2 A_2 A_3}{189} + \frac{A_4^2 A_3^2}{405} - \frac{2A_3 A_4 F_0}{45}\Bigg)
\end{aligned} \tag{3.5}$$

where

$$F_0 = \frac{1}{2\left\{1+\frac{3}{a^2}\left(\frac{\tanh a}{a}-1\right)\right\}}\left(\begin{array}{l}\frac{Y^2}{2}-\frac{Y^4}{4}+\frac{6}{a^2}(\frac{\cosh aY}{a^2\cosh a} \\ -\frac{\tanh a}{a}\frac{Y^2}{2})+F-12\sum_{n=1}^{\infty}e^{-\lambda_n{}^2\tau} \\ \frac{Cos\lambda_n Y}{\lambda_n{}^2}\left(\frac{1}{\lambda_n{}^2}-\frac{\tanh a}{a(\lambda_n{}^2+a^2)}\right)\end{array}\right)$$

and $F=-\left(\frac{7}{60}+\frac{6\tanh a}{a^5}-\frac{\tanh a}{a^3}\right)$ (3.6)

We note that Eq. (3.3) has no physical meaning as it stands. We, however, found that $K_3(\tau), K_4(\tau)$ and so on are all zero's.

Thus $\frac{\partial\theta_m}{\partial\tau}=K_2\frac{\partial^2\theta_m}{\partial X^2}$ (3.7)

which is the usual diffusion equation.

This form, though similar to the Taylor's (1953) model, differs from it due to the fact that the most dominant dispersion coefficient $k_2(\tau)$ is time dependent.

It will be useful if we have an estimate of the separate contributions of diffusion and pure convection on dispersion coefficient. To that end we now evaluate the contribution of pure convention, the result of which cannot be obtained from the above general case. Following the earlier procedure $k_2(\tau)$ for pure convection (i.e. neglecting the diffusion term in equation (3.5), is given by

$$K_2(\tau)=\frac{\tau}{72A_6^2}\left(\begin{array}{l}\frac{36}{a^4\cosh^2 a}-\frac{\left(48a^2+108\right)}{a^5}\tanh a-\frac{72}{a^6}\tanh^2 a+\frac{8}{5a^4}(a^4+90) \\ +A_4^2A_5^2\left(\frac{\sinh 2a}{2a}-1\right)+\frac{4A_4^2A_2A_5}{a^2}\left(\sinh a-a\cosh a\right)+\frac{4A_4^2A_3A_5}{a^4}(6\sinh a-6a\cosh a \\ +3a^2\sinh a-a^3\cosh a)+\frac{2A_4^2A_2^2}{3}+\frac{4A_4^2A_2A_3}{5}+\frac{2A_4^2A_3^2}{7}\end{array}\right)$$

(3.8)

The exact solution is given by

$$\theta_m(\xi,\tau)=\frac{1}{2}[(\frac{\frac{X_s}{2}+\xi}{2\sqrt{T}})+erf(\frac{\frac{X_s}{2}-\xi}{2\sqrt{T}})]$$

Where, $T=\int_0^\tau K_2(z)dz$, and $erf(x)=\frac{1}{\pi}\int_0^x e^{-z^2}dz$ (3.9)

4. Results and Discussions

Using generalized dispersion model of Gill and Sankarasubramanian (1970) the unsteady convective diffusion in a couple stress poorly conducting fluid bounded by impermeable rigid boundaries is studied. The most dominant dispersion co-efficient given by (3.5) is computed for different values of couple stress parameter ‘a’, electric number ‘We’ and the dimensionless time τ . The results are graphically represented in figures (1) and (2). The parameter ‘a’ which depends on the size of suspended particles in physiological problems, greatly influences the dispersion co-efficient because it can be regarded as the characterization of the interaction of fluid particles with the geometry of the channel. From fig. 1 we note that the effect of couple stresses on the dispersion co-efficient is prominent for small values of ‘a’ as the small values of ‘a’ corresponds to either to a suspended molecule with a long chain or to a small width of the channel. The latter is important to explain the rheological abnormalities of blood flow through a channel of small width. It shows that the dispersion coefficient decreases with an increase in couple stress parameter ‘a’ and increases with an increase in electric number ‘We’. Initially K_2 increases gradually up to the value $\tau = 1$ and remains uniform for values of τ greater than 1. In this figure, the result for We = 0 corresponds to those given by Rudraiah et al (1986) and result for $\sigma\rightarrow\infty$ corresponds to Rudraiah et al (2011) for permeable boundaries.

The transport of major metabolizes (such as sugars and amino acids) is rather slow and convective transport plays a major role in accelerating them. Therefore, we have computed θ_m given by (3.5) and the results of the variation of mean concentration θ_m with axial distance x for a fixed τ are plotted in figures (3) and (4). These figures reveal a marked variation of θ_m with time and the effect of ‘a’ is to decrease the concentration distribution, a result that is true for combined convection plus diffusion (C+D) and pure convection(C).Figures (5), (6), (7) and (8) represent the variation of mean concentration θ_m of a tracer along the pressure gradient, at a given point, with τ for different values of ‘a’, ‘We’ and for a fixed x. In figures (5) and (6), the observation point is inside the concentration slug whereas in figures (7) and (8), it is outside. In figures (5) and (6) the observation point is close to the entrance and the influence of ‘a’ and ‘We’ on θ_m is negligible, also pure convection has a major contribution to θ_m. From figures (7) and (8), it is clear that the effect of pure convection is decreased by the same magnitude when the observation point gets farther and farther away from the entrance. We see that the dispersion (molecular diffusion and convection) is faster, i.e. the parabolic nature of θ_m with τ at a fixed x/h if the observation point is far away from the entrance. In these figures we note that θ_m for convection and diffusion(C+D) and θ_m for pure convection(C) increase with a decrease in ‘a’ and ‘We’ and for small values of τ, θ_{mcd} curve is above θ_{mc} . In other words the couple stress parameter significantly influences θ_m .

Conclusions:

(i) The couple stress are operative only for small values of ‘a’ and the present results reduce to Newtonian fluids in the limit of $a\rightarrow\infty$.

(ii) Taylor’s dispersion model(1953) forms a particular case of the generalized dispersion model for asymptotic values of τ.

(iii) Results of Rudraiah et al (1986) form a particular case of the present study for We=0 and for large values of σ , the results of Rudraiah et al (2011) reduce to those of present study.

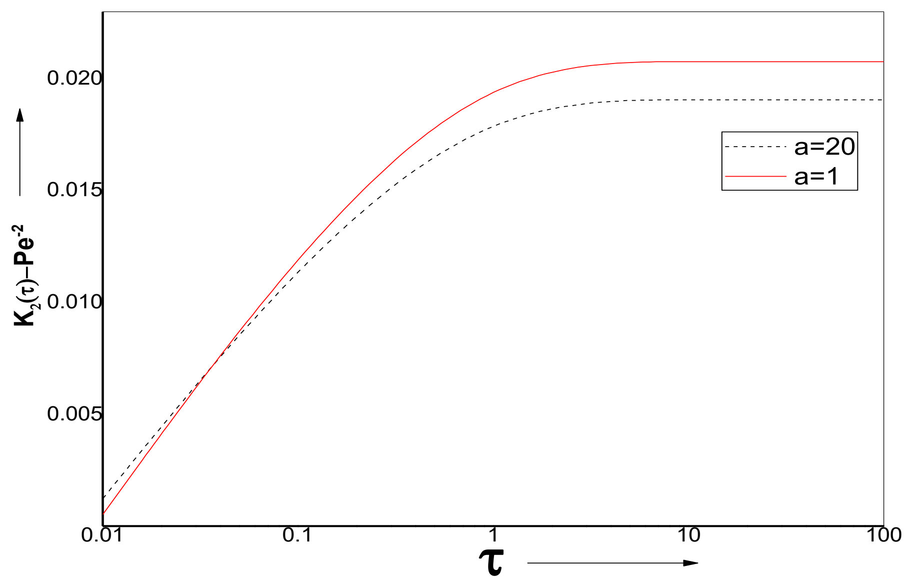


Fig.1: Effect of couple strees on unsteady dispersion coefficient

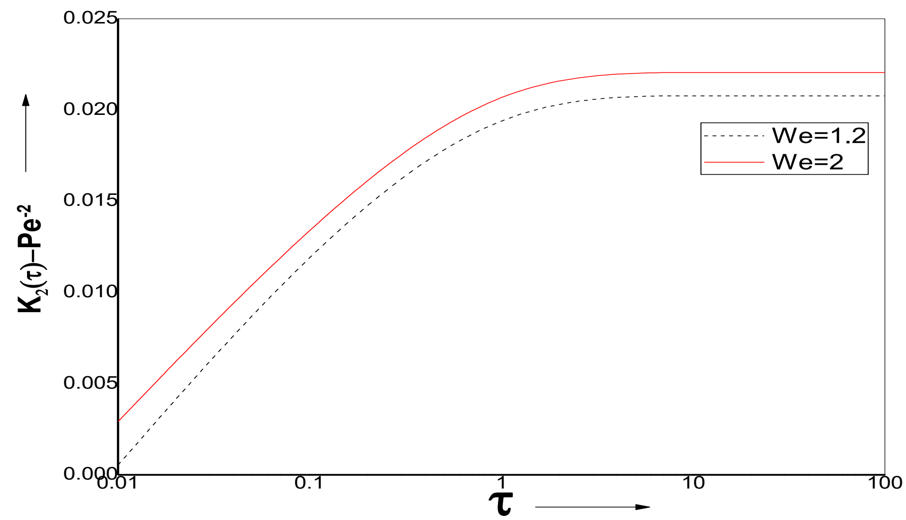


Fig.2: Effect of electric no. (We) on unsteady dispersion coefficient

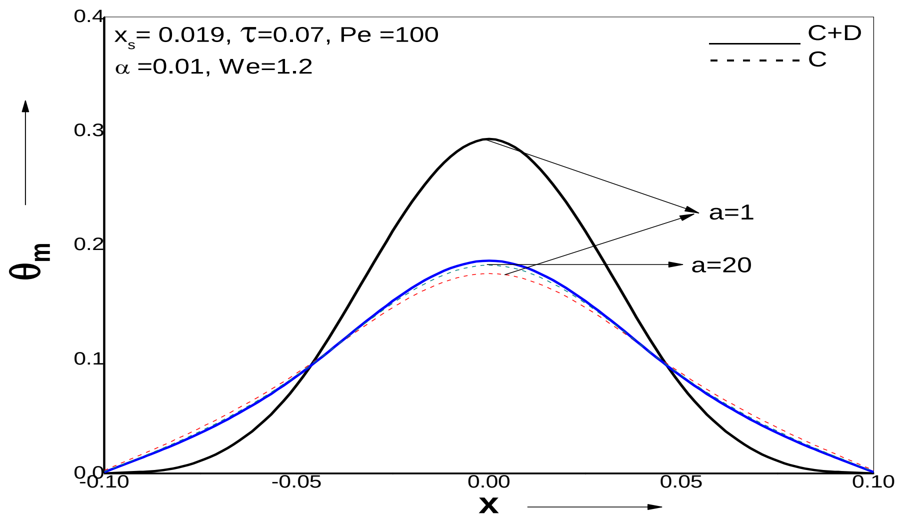


Fig. 3: Plots of θ_m verses x for different values of a

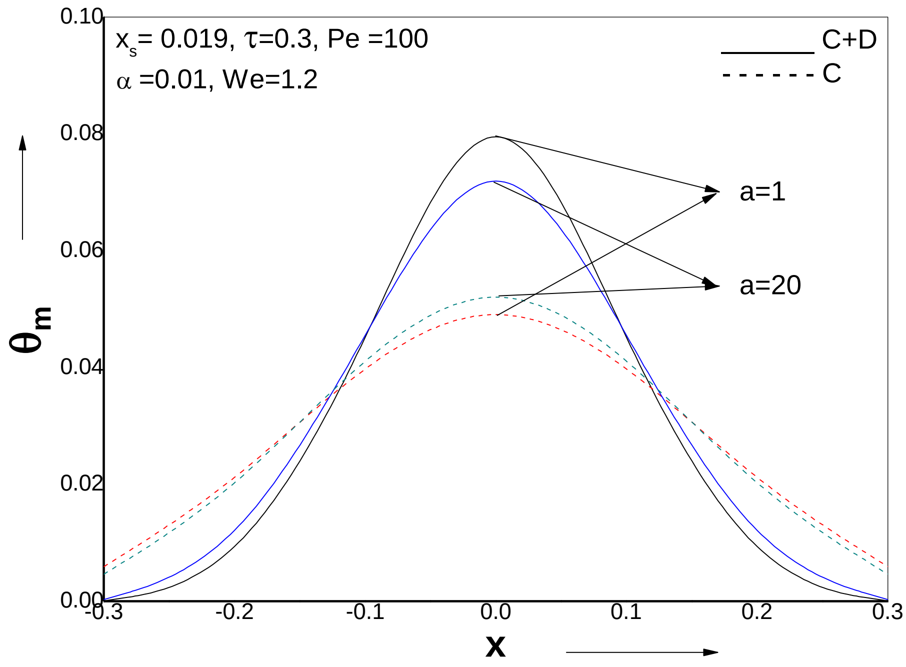


Fig. 4: Plots of θ_m verses x for different values of a

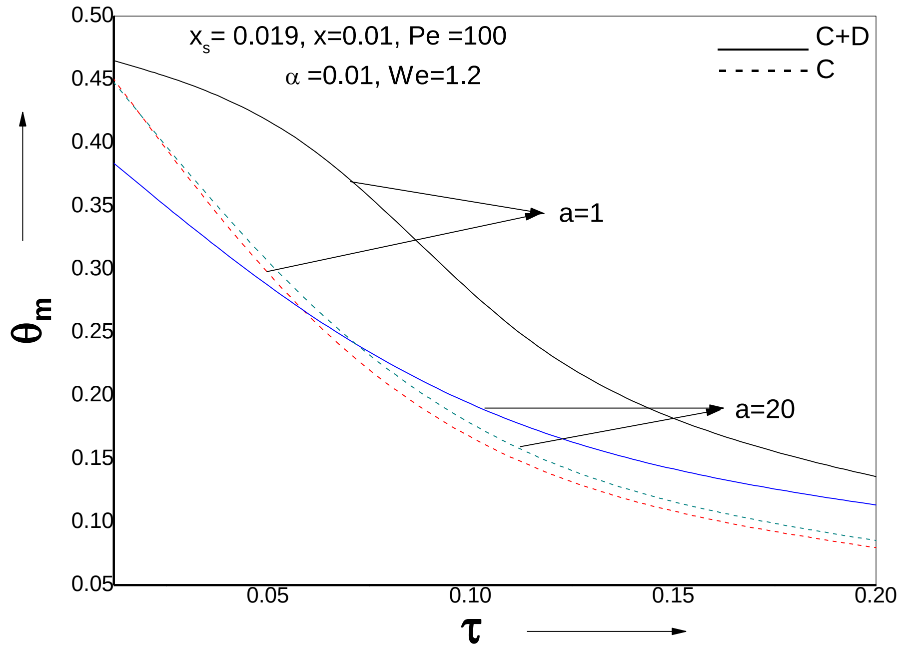


Fig. 5: Plots of θ_m verses τ for different values of a

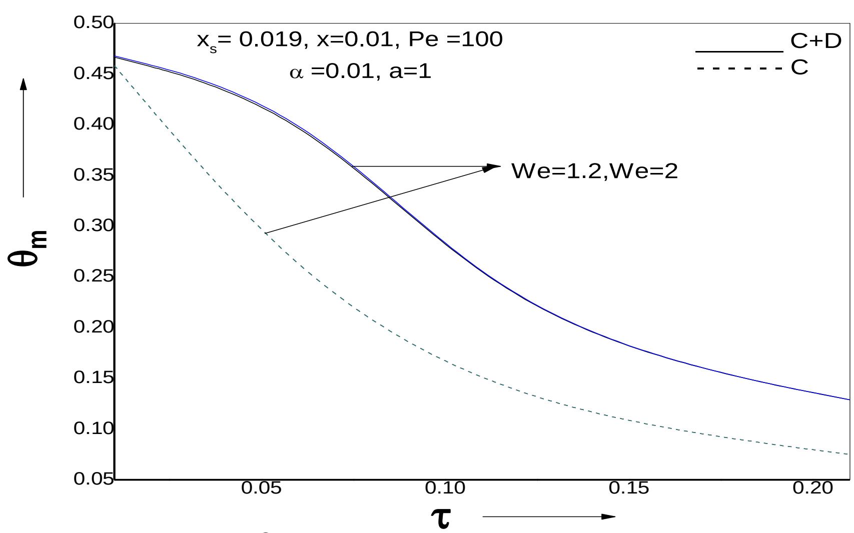


Fig. 6: Plots of θ_m verses τ for different values of We

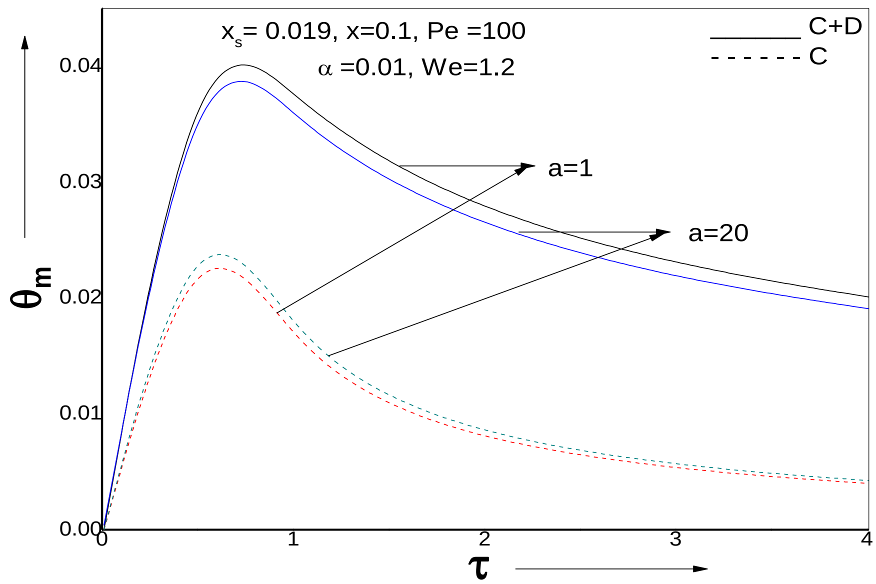


Fig. 7: Plots of Mean θ_m verses τ for different values of a

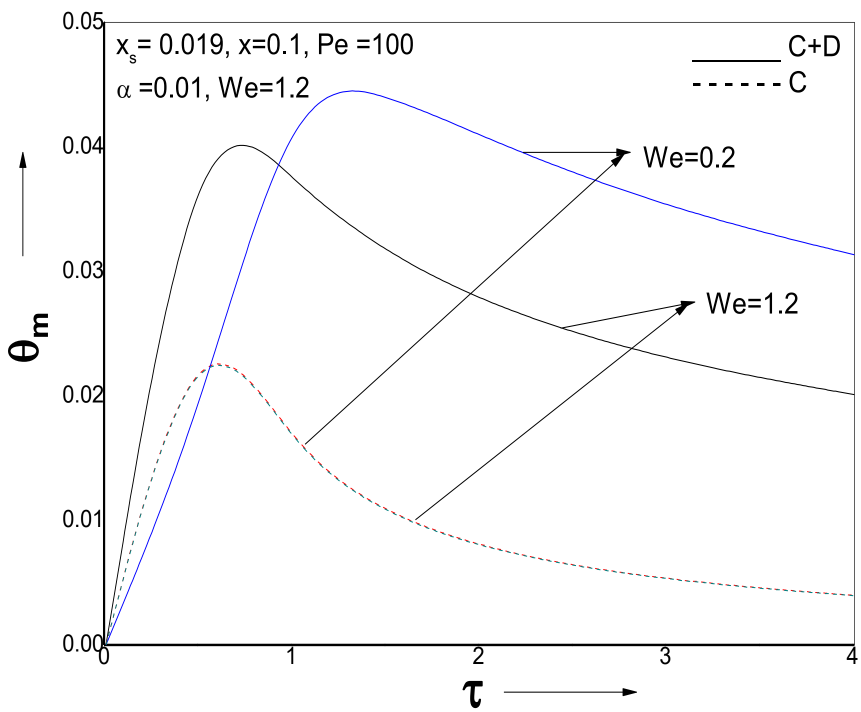


Fig. 8: Plots of θ_m verses τ for different values of We

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