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# MONOPHONIC WIENER INDEX OF A GRAPH

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***Abstract***
*The main purpose of this paper is to investigate Monophonic Wiener Index of certain graphs using Monophonic distance matrix.*

***Key Words: Monophonic Path, Monophonic Number, Weiner Index.***

## 1. Introduction

In this paper G denotes (G = (V, E)) a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by n and m respectively. For basic definitions and terminologies and for the concepts of distance in graph we refer to [5, 8, 13].

A Chord of a path $u_0, u_1, u_2, \cdots, u_h$ is an edge $u_i u_j$ with $j \geq i+2$. A $u - v$ path is called a monophonic path if it is a chordless path. The monophonic distance is studied in [10, 11].

Wiener Index was introduced by Harold Wiener in 1947. Wiener Index has been used to model various properties of chemical species. Chemical graphtheory is used to model physical properties of molecules called alkanes. Alkanes are organic compounds composed of carbon and hydrogen atoms. For this refer [6]. A molecular graph is a collection of points called the atoms in the molecule and the set of lines as the covalent bounds .Thus a vertex will be defined by an atom and the edge is defined by a given bound in molecule. Wiener Index of a graph has undergone several developments such as modified Wiener Index ,Hyper Wiener Index and Variable Wiener Index etc. The modified Wiener Index gives greater contribution to outer bounds than to inner bounds of a molecule. For this refer [1, 2, 3, 4, 7, 9, 10, 12]. Wiener Index was also successfully tested in several structures -property relationship, so it is very meaningful to research their mathematical properties and chemical application. This is the motivation behind the introduction and study of Weiner Index using Monophonic and detour distance concepts.

### 1.1. Notation and Terminology

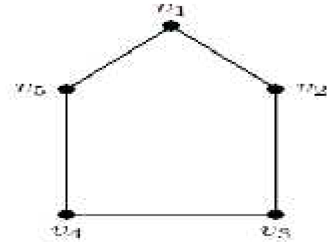
We consider finite and simple graphs and use standard terminology. For a graph G, the vertex set is denoted V (G) and the edge set is denoted $E(G)$. If $e = \{u, v\}$ an edge of a graph G, we write $e = uv$, we say that e joins the vertices $u$ and $v$; $u$ and $v$ are adjacent vertices; $u$ and $v$ are incident with *e* . If two vertices are not joined, then we say that they are non-adjacent. If two distinct edges *e* and *f* are incident with a common vertex $v$, then *e* and *f* are said to be adjacent to each other. A set of vertices in a graph is independent if no two vertices in the set are adjacent. A vertex of degree 0 in *G* is called an end-vertex of *G*.*A* cut-vertex (cut-edge) of a graph *G* is a vertex (edges) whose removal increases the number of components. A vertex $v$ is an extreme vertex of a graph *G* if the subgraph induced by its neighbors is complete.

## 2. Preliminaries

In this section, we have given definitions, example and a theorem which will be used in our main results.

**2.1. Definition:** Monophonic distance matrix of a graph *G* is defined as a square matrix $D_m(G) = [d_{m_{ij}}]$, where $d_{m_{ij}}$ is the length of the longest monophonic path between the vertices $v_i$ and $v_j$ in $G$.

**2.2. Example:** Monophonic distance matrix of the graph *G* is given below.




$C_5$
Figure 2.1

$$\begin{array}{c|c} & \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ D_m(G) = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{pmatrix} 0 & 1 & 3 & 3 & 1 \\ 1 & 0 & 1 & 3 & 3 \\ 3 & 1 & 0 & 1 & 3 \\ 3 & 3 & 1 & 0 & 1 \\ 1 & 3 & 3 & 1 & 0 \end{pmatrix} \end{array}$$

**2.3. Definitions:** The Wiener index *W (G)* of a graph *G* is defined as the sum of the half of the distances between every pair of vertices of *G*.

$$WI(G) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d(u_i, v_j).$$
$$= \sum_{i<j} d(u_i, v_j).$$

**3. Monophonic Wiener Index of a Graph**
**3.1. Definitions:** The Monophonic Wiener Index of a graph *G* is denoted by *Wm I (G)* and is defined as

$$W_mI(G) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}[D_m(G)]_{ij}$$
$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{m_{ij}}$$
$$= \sum_{i<j} d_m(v_i, v_j)$$

**3.2. Example:** Monophonic Wiener Index of a graph *G* in Figure 3.1 is given below.

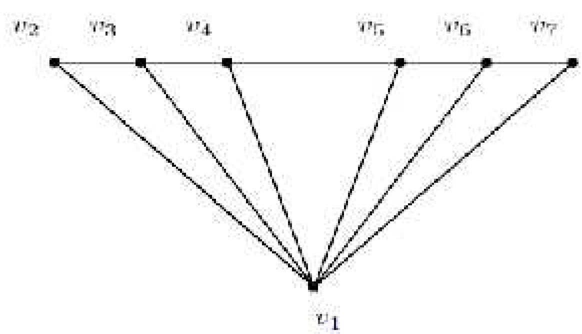



Figure 3.1

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$$
\begin{array}{c c}
 & \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\
D_m(G) = \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} &
\begin{pmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 \\
1 & 1 & 0 & 1 & 2 & 3 & 4 \\
1 & 2 & 1 & 0 & 1 & 2 & 3 \\
1 & 3 & 2 & 1 & 0 & 1 & 2 \\
1 & 4 & 3 & 2 & 1 & 0 & 1 \\
1 & 5 & 4 & 3 & 2 & 1 & 0
\end{pmatrix}
\end{array}
$$

$$
\begin{aligned}
W_mI(G) &= \frac{1}{2}\sum_{i=1}^{7}\sum_{j=1}^{7}[D_m(G)] \\
&= \frac{1}{2}\sum_{i=1}^{7}\sum_{j=1}^{7} d_m(v_i, v_j) \\
&= \sum_{i<j} d_m(v_i, v_j) \\
&= (6 + (6-1)1 + (6-2)2 + (6-3)3 + (6-4)4 + (6-5)5) \\
&= 41.
\end{aligned}
$$

**Theorem 3.3:** Monophonic Wiener Index of the Cycle graph *Cn* (*n* is odd) is

$$
W_mI(C_n) = \begin{cases} n & \text{if } n = 3 \\ n + \sum_{k=2}^{\frac{n-1}{2}}[n(n-k)] & \text{if } n > 3 \end{cases}
$$

Proof. Let the vertices of the Cycle $C_n$ be $v_1, v_2, \cdots, v_n$ such that $v_1$ is adjacent to $v_n$, $v_i$ is adjacent to $v_{i+1}, 1 < i < n-1$.

Case 1: For *n*>3

$$
\begin{aligned}
W_mI(C_n) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}[D_m(C_n)]_{ij} \\
&= \sum_{\substack{v_i, v_j \in V(G) \\ i<j}} d_m(v_i, v_j) \\
&= n(n-2) + n(n-3) + \ldots + n\left(n - \left(\frac{n-1}{2}\right)\right) + n \\
&= n + n\left((n-2) + (n-3) + \ldots + n - \left(\frac{n-1}{2}\right)\right)
\end{aligned}
$$

$$
W_mI(C_n) = n + \sum_{k=2}^{n-1} n(n-k), n > 3
$$

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Case 2: For $n$ =3
Since $d_m(v_i, v_j) = 1, 1 \leq i < j \leq 3.$

$$W_mI(C_3) = \sum_{i<j} d_m(v_i, v_j) = 3$$

**Theorem 3.4:** Monophonic Wiener Index of the Cycle graph $Cn$ ($n$ is even) is

$$W_mI(C_n) = \begin{cases} 2n & \text{if } n = 4 \\ n + \sum_{k=2}^{\frac{n-2}{2}} n(n-k) + \left(\frac{n}{2}\right)^2 & \text{if } n > 4 \end{cases}$$

Proof. Let $v_1, v_2, v_3, \ldots v_n$ the n vertices of the Cycle graph $Cn$ ($n$ is even) such that $v_1$ is adjacent to $v_n$ and $v_i$ is adjacent to $v_{i+1}, 1 \leq i \leq n-1.$

Case 1: For $n$>4
By definition,

$$\begin{aligned} W_mI(C_n) &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}[D_m(C_n)]_{ij} \\ &= \sum_{\substack{v_i, v_j \in V(G) \\ i<j}} d_m(v_i, v_j) \\ &= (n-1)+1+(n-2)(n-2)+2(n-2)+\ldots+n\left(n-\left(\frac{n-2}{2}\right)\right) \\ &+ \left(\frac{n}{2}\left(n-\left(\frac{n}{2}\right)\right)\right) \\ &= \left(n+n(n-2)+n(n-3)+\ldots+n\left(n-\left(\frac{n-2}{2}\right)\right)\right)+\left(\frac{n}{2}\right)^2 \\ &= n+\sum_{k=2}^{\frac{n-2}{2}} n(n-k)+\left(\frac{n}{2}\right)^2, n>4 \end{aligned}$$

Case 2: For $n$=4
Since

$$d_m(v_i, v_j) = \begin{cases} 1 & \text{if } i \text{ is adjacent to } j \\ 2 & \text{if } i \text{ is not adjacent to } j \end{cases}$$

$$\begin{aligned} W_mI(C_4) &= \sum_{i<j} d_m(v_i, v_j) \\ &= 8 = 2n \end{aligned}$$

**Theorem 3.5:** Monophonic Wiener Index of the fan graph $fn$, $n \geq 2$ is

$$W_mI(f_n) = \begin{cases} 2n-1 & \text{if } n = 2 \\ 2n-1+\sum_{k=2}^{n-1}(n-k)k & \text{if } n \geq 3 \end{cases}$$

Proof. Let $v_1, v_2, v_3, \ldots, v_n, v_{n+1}$ be the $n+1$ vertices of the fan graph $f_n$. Then $f_n = P_n + K_1$

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Let $K_1 = v_1$ and $v_1$ adjacent to the vertices $v_2, v_3, \dots, v_n, v_{n+1}$.
Then $d_m(v_1, v_i) = 1, 2 \le i \le n+1, d_m(v_i, v_{i+1}) = 1, 2 \le i \le n$ and $d_m(v_i, v_{i+j}) \ge 2, 2 \le i \le n-1, 2 \le j \le n, i+j \le n+1$

Case 1: For $n \ge 3$

$$W_m I(f_n) = \frac{1}{2} \sum_{j=1}^{n+1} \sum_{i=1}^{n+1} [D_m(f_n)]_{ij}$$

$$= \sum_{\substack{v_i, v_j \in V(G) \\ i<j}} d_m(v_i, v_j)$$

$$= n + (n-1)1 + (n-2)2 + (n-3)3 + \dots + (n-(n-1))(n-1)$$

$$W_m I(f_n) = 2n - 1 + \sum_{k=2}^{n-1} (n-k)k, n \ge 3$$

Case 2: For $n=2$

$$d_m(v_i, v_j) = \begin{cases} 1 & \text{if } i \ne j \\ 0 & \text{if } i = j \end{cases}$$

Since

$$W_m I(f_2) = \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} [D_m(f_2)]_{ij}$$

$$\frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} d_m(v_i, v_j)$$

$$= 2n - 1, n = 2$$

**Theorem 3.6.** The Monophonic Wiener Index of the Friendship graph $F_n$ is $W_m I(F_n) = (2n)^2 - n, n \ge 2$.

Proof. Let $v_1, v_2, v_3, \cdots v_{2n+1}$ be the $2n+1$ vertices of the Friendship graph $Fn$.
$Fn$ has $n$ number of triangles with a common vertex $v_1$ and it has $3n$ edges.
Then

$$d_m(v_1, v_j) = 1, 2 \le j \le 2n+1.$$

$$d_m(v_i, v_j) = \begin{cases} 1 & \text{if } i \text{ is adjacent to } j \\ 2 & \text{if } i \text{ is not adjacent to } j \end{cases}$$

By definition,

$$W_m I(F_n) = \frac{1}{2} \sum_{j=1}^{2n+1} \sum_{i=1}^{2n+1} [D_m(F_n)]_{ij}$$

$$= \sum_{j=2}^{2n+1} d_m(v_1, v_j) + \sum_{2 \le i < j \le 2n+1} d_m(v_i, v_j)$$

$$= [2n + n + 2n(n-1)2]$$
$$W_m I(F_n) = ((2n)^2 - n), n \geq 2.$$

**Theorem 3.7** Monophonic Wiener Index of the Bistar graph $B_{m,n}$ is $W_m I(B_{m,n}) = (m+n+1)^2 + mn$.

Proof. Le $u, v$ be the vertices of $K_2$ in Bistar graph $B_{m,n}$.
Let $u = \{u_1, u_2, u_3, \cdots, u_m\}$ and $v = \{v_1, v_2, v_3, \cdots, v_n\}$ be the set of vertices adjacent to $u$ and $v$ respectively.
In Bistar graph,
$d_m(u_i, u) = 1, 1 \leq i \leq m, d_m(u_i, v) = 2, 1 \leq i \leq m.$
$d_m(v, v_j) = 1, 1 < j < s, d_m(u, v_j) = 2, 1 < j < s.$
$d_m(v, u) = 1, d_m(u_i, u_k) = 2, 1 \leq i < k \leq m.$
$d_m(v_j, v_k) = 2, 1 \leq j < k \leq n, d_m(u_i, v_j) = 3, 1 \leq i \leq m, 1 \leq j \leq n.$

The Monophonic Wiener Index of the Bistar graph $B_{m,n}$ is

$$\begin{aligned} W_m I(B_{m,n}) &= \sum_{i=1}^{m} d_m(u_i, u) + \sum_{j=1}^{n} d_m(v, v_j) + d_m(u, v) + \sum_{i=1}^{m} d_m(u_i, v) + \sum_{j=1}^{n} d_m(u, v_j) \\ &+ \sum_{1 \leq i < k \leq m} d_m(u_i, u_k) + \sum_{1 \leq j < k \leq n} d_m(v_j, v_k) + \sum_{i=1}^{m}\sum_{j=1}^{n} d_m(u_i, v_j) \\ &= m + n + 1 + (2m) + (2n) + \binom{m(m-1)}{2}(2) + \binom{n(n-1)}{2}(2) + 3mn \\ &= (m+n+1)^2 + mn \end{aligned}$$

**Theorem 3.8.** The Monophonic Wiener Index of the Wheel graph $W_n$ ($n$ is odd) is

$$W_m I(W_n) = \begin{cases} 2n & \text{if } n = 3 \\ 2n + \sum_{k=3}^{\frac{n+1}{2}} n(n+1-k) & \text{if } n > 3 \end{cases}$$

Proof. Let the vertices of the Wheel graph $W_n$ be $v_1, v_2, v_3, \cdots, v_n, v_{n+1}$. Then $W_n = C_n + K_1$.
Then $W_n = C_n + K_1$.

Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the Cycle $C_n$.

Let $K_1 = v_{n+1}$ be the center vertex of $W_n$.

The verte: $v_i$ is adjacent to $v_{i+1}, 1 \leq i \leq n-1$.

Let $v_1$ is adjacent to $v_n$ and $v_{n+1}$ is adjacent to $v_1, v_2, v_3, \cdots, v_n$.

Case 1: For $n > 3$.

$$\begin{aligned} W_m I(W_n) &= \frac{1}{2}\sum_{i=1}^{n+1}\sum_{j=1}^{n+1} |D_m(W_n)|_{ij} \\ &= \sum_{\substack{v_i, v_j \in v(G) \\ i<j}} d_m(v_i, v_j) \\ &= n + n + n(n+1-3) + n(n+1-4) + \cdots + n\left(n+1-\binom{n+1}{2}\right) \end{aligned}$$

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$$= 2n + n(n+1-3) + n(n+1-4) + \cdots + n\left(n+1-\left(\frac{n+1}{2}\right)\right)$$

$$W_mI(W_n) = 2n + \sum_{k=3}^{\frac{n+1}{2}} n(n+1-k)$$

Case 2: For $n = 3$

Since

$$d_m(v_i, v_j) = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$$

$$W_mI(W_3) = 6 = 2n.$$

**Theorem 3.9.** The Monophonic Wiener Index of the Wheel graph $W_n$ ($n$ is even) is

$$W_mI(W_n) = \begin{cases} 2(n+2) & \text{if } n = 4 \\ 2n + \sum_{k=3}^{\frac{n}{2}} n(n+1-k) + (\frac{n}{2})^2 & \text{if } n > 4 \end{cases}$$

Proof. Let the vertices of the Wheel graph $Wn$ be $v_1, v_2, v_3, \cdots, v_{n+1}$.

Then $W_n = C_n + K_1$.

Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the Cycle $C_n$.

Let $K_1 = v_{n+1}$ be the center vertex of $W_n$.

The vertex $v_i$ is adjacent to $v_{i+1}, 1 \leq i \leq n-1$.

$v_1$ is adjacent to $v_n$ and $v_{n+1}$ is adjacent to $v_1, v_2, v_3, \cdots, v_n$.

Case 1: For n > 4.

$$W_mI(W_n) = \frac{1}{2}\sum_{i=1}^{n+1}\sum_{j=1}^{n+1}[D_m(W_n)]_{ij}$$

$$= \sum_{\substack{v_i, v_j \in v(G) \\ i<j}} d_m(v_i, v_j)$$

$$= n + n + n + (n+1-3)n(n+1-4) + \cdots + n\left(n+1-\left(\frac{n}{2}\right)^2\right)$$

$$= 2n + \sum_{k=3}^{\frac{n}{2}} n(n+1-k) + \left(\frac{n}{2}\right)^2$$

$$W_mI(W_n) = 2n + \sum_{k=3}^{\frac{n}{2}} n(n+1-k) + \left(\frac{n}{2}\right)^2$$

Case 2: For $n = 4$

$$d_m(v_i, v_j) = \begin{cases} 1 \text{ if } i \text{ is adjacent to } j \\ d_{m_{ij}} \text{ if } i \text{ is not adjacent to } j \end{cases}$$

$$W_mI(W_4) = \frac{1}{2}\sum_{i=1}^{5}\sum_{j=1}^{5}[D_m(W_4)]_{ij}$$
$$= 2(4+2) = 2(n+2).$$

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**Theorem 3.10.** The Monophonic Wiener Index of the Crown graph $C_{n,n}$ is

$$W_mI(C_{n,n}) = \begin{cases} n & \text{if } n = 2 \\ 5n^2 - 2n & \text{if } n > 2 \end{cases}$$

Proof. Let $v_1, v_2, v_3, \cdots, v_{2n}$ be the vertices of the crown graph $C_{n,n}$ also it has $2_n$ edges.
Case 1: For $n = 2$

$$d_m(v_i, v_j) = \begin{cases} 1 \text{ if } i \text{ is adjacent to } j \\ d_{m,}, \text{if } i \text{ is not adjacent to } j \end{cases}$$

Since $d_m(v_1, v_1) = 0, d_m(v_1, v_2) = 0, d_m(v_1, v_3) = 0, d_m(v_1, v_4) = 1, d_m(v_2, v_2) = 0,$
$d_m(v_2, v_3) = 1, d_m(v_2, v_4) = 0, d_m(v_3, v_3) = 0, d_m(v_3, v_4) = 0, d_m(v_4, v_4) = 0,$

$$W_mI(C_{2,2}) = 2 = n.$$

Case 2: For $n > 2$
By definition,

$$\begin{aligned} W_mI(C_{n,n}) &= \frac{1}{2}\sum_{j=1}^{2n}\sum_{i=1}^{2n}[D_m(C_{n,n})]_{ij} \\ &= \sum_{\substack{v_i, v_j \in v(G) \\ i<j}} d_m(v_i, v_j) \\ &= 4n(n-1) + 3n + n(n-1) \\ &= 5n^2 - 2n \end{aligned}$$

**Theorem 3.11.** The Monophonic Wiener Index of the Gear graph $G_n$ ($n \geq 3$) is

$$W_mI(G_n) = n^2 + 5n + \sum_{k=1}^{n-2} 2n(n+k)$$

Proof. Let $v_1, v_2, v_3, \cdots, v_{2n-1}$ be the $2n-1$ vertices of the Gear graph $G_n$ and also it has $3_n$ edges. Let $v_1, v_2, v_3, \cdots, v_{2n}$ be the vertices of the Cycle $C_n$ and $v_{2n+1}$ be the center vertex of the Gear Graph.

$$\begin{aligned} W_mI(G_n) &= \frac{1}{2}\sum_{j=1}^{2n+1}\sum_{i=1}^{2n+1}[D_m(G_n)]_{ij} \\ &= \sum_{i<j} d_m(v_i, v_j) \\ &= 5n + n^2 + [2n(n+1) + 2n(n+2) + 2n(n+3) + \cdots + 2n(n+(n-2))] \\ &= 5n + n^2 + \sum_{k=1}^{n-2} 2n(n+k) \end{aligned}$$

**Theorem 3.12.** If $G$ is a connected graph with $n \geq 2$ then $1 \leq WI(G) \leq WmI(G)$.
Proof. Let $\{v_1, v_2\}$ be the two vertices of $G$, where $G = P_2$ *or* $K_2$.
Their monophonic distance is equal to its shortest distance. (i.e) $D(G) = Dm(G)$ and so $WI(G) = WmI(G) = 1$.
Moreover for a graph $G$ other than $P_2$ *or* $K_2$, $d(v_i, v_j) > 1$, where $v_i, v_j \in v(G)$
Therefore $1 \leq WI(G) \leq W_m I(G)$.

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**Result 3.13.** The bounds in the above theorem is sharp. For the graph $G = C3$ *or* $K3$, $W\,I\,(G) = Wm\,I\,(G)$
For the graph $G = P2$ *or* $K2$, $W\,I\,(G) = 1$.
Also the inequality in this theorem is strict.
For the Wheel graph $G = C5$, $W\,I\,(G) = 15$, $Wm\,I\,(G) = 20$. Thus $W\,I\,(G) < Wm\,I\,(G)$
For the graph G = $f2$, $W\,I\,(G) = 3$
Thus $1 < W\,I\,(G)$
Therefore $1 \leq W\,I\,(G) \leq Wm\,I\,(G)$.

**References**

1. Baskar Babujee.J and J. Senbagamalar, Wiener Index of Graphs using Degree Sequence, Applied Mathematical Sciences,Vol.6(2012), no.88, 4387-4395.
2. Dankelmann. p,I. Gutman, S. Mukwembi, H.C Swart, On the degree distance of a graph, Discrete Mathematics,Vol. 157, No. 13,(2009),2773-2777.
3. Dobrynin.A.A,Entringer.R, Wiener Index of Trees: Theory and Applications, Acta Appli-candae Mathematicae, Vol. 66, No. 3, (2001), 211-249.
4. Eliasi.M, B.Teari, Four new sums of graphs and their Wiener Indices, Discrete Applied Mathematics.157 (2009) 794-803.
5. Gary Chartrand and Ping Zhang, Distance in Graphs Taking the Long View, AKCE.J. Graphs, combin, (2004) 1(1), 1-13.
6. Huheey, James E, Ellen. A, Richard L, Inorganic Chemistry, Principles of structure and reactivity, 4(1993), 256-280.
7. Ivan Gutman and Janez, Zerovnik, Class of Modified Wiener Indices, CROATICA CHEMICA ACTA, CCACAA 77 (12), (2004) 103-109.
8. Murugan. M,Graph Theory and Algorithms, Discrete Mathematics,(2003),1-4,20-21.
9. Mohar.B and T.Pisanski, How to Compute the Wiener Index of a Graph, Journal of Mathematical Chemistry, 2(1988)267-277.
10. Santhakumaran A.P and Titus. P, A note on Monophonic Distance in Graphs, Discrete Math. Algorithms Appl.4 (2), (2012), 1-5.
11. Santhakumaran A.P and Titus.P, Monophonic Distance in Graphs, Discrete Math Algorithms Appl.3 (2) (2011), 159-169.
12. Stevanovic, Maximizing Wiener Index of Graphs with fixed maximum degree, Communications in Mathematical and Computer Chemistry, (2008).
13. West D. B, Introduction to graph Theory, (2004), Prentice-Hall of India, New Delhi.