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FLOW OF DUSTY GAS UNDER TRANSVERSE MAGNETIC FIELD AND SHOCK WAVES IN THE ATMOSPHERE OF A ROTATING STAR

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Introduction

In the present communication we have studied flow the dusty gas as well as unsteady fluid and dust particles. In the first part the flow of dusty gas in three dimensions under transverse magnetic field has been studied. Both types i.e. steady and unsteady plane parallel flows are considered separately. It has been assumed that gas viscosity in constant and the distribution of dust particles in the gas is uniform. In both the cases of steady and unsteady motion it has been observed that phase angles of magnetic lives are orthogonal to those of streamlines of the dusty gas flow. The paths are found to a parabolic. Velocity of gas and dust particles with satisfies the same differential equation of unsteady flow. We have carried out the analysis of flow in both osculating and rectifying planes. It has been shown that the flow reduces to unidirectional flow and dust particles have the same velocity as velocity of gas. We have got expressions for these velocities in terms of curvature and torsions of streamlines and the strength of the magnetic lines.

As a result of unsteady processes at photosphere levels, researchers are now a days of the opinion that there is shock wave propagation in the solar chromospheres and probably into the corona too and so we have studies the behavior of shock waves in an atmosphere of rotating star under the force of gravitation in the second part of the present communication.

For the purpose of study, the well known method of Witham is used and the variation in density strength of the spiracle shock has been investigated in a rotating gravitating atmosphere with arbitrary temperature distribution, and steady mass motions. Particular cases of very strong and weak shocks have been discussed in detail. Gradient of temperature and mach number in case of steady mass motion have been derived and discussed.

2. Steady Flow of Dusty Gas

A steady dusty-gas flow is governed by the following reduced system of equations:

$$\mathrm{div}(\rho\vec{u}) = 0 \qquad \text{……… (2.1)}$$

$$(\vec{u}\nabla)\vec{u} = -\rho^{-1}\nabla p + v\nabla^2\vec{u} + \frac{f}{\tau}(\vec{v}-\vec{u}) - \sigma B_0^2\vec{u} \qquad \text{……… (2.2)}$$

$$\mathrm{div}(N\vec{v}) = 0 \qquad \text{……… (2.3)}$$

$$(\vec{v}.\nabla)\vec{v} = \frac{K}{m}(\vec{u}-\vec{v}) \qquad \text{……… (2.4)}$$

where $\vec{u}$ and $\vec{v}$ denote respectively fluid velocity and dust velocity, ρ stands for density of gas, p is pressure of gas, $v = \frac{\mu}{\rho}$ is kinematic velocity, μ is viscosity of gas, $\tau = \frac{m}{K}$ is non dimensional relation time, m is mass of dust particles, k stands for Stroke's residence coefficient for spherical particles and equals to 6πaμ for dust particles of radius a, $f = \frac{mN}{P}$ is mass acceleration, N is number of dust particular and B_0 stands for the strength of magnetic field.

Further the following restrictions are imposed on the flow

a. Magnetic field lines are perpendicular to stream velocity.
b. Gas viscosity is constant i.e. $\mu \propto \rho$.
c. The dust particles are uniformly distributed, in the gas.
d. Velocity of the gas $\vec{u}$ is parallel to the velocity of the dust $\vec{v}$ in particular, we take $\vec{u} = u\vec{s}$ and $\vec{v} = v\vec{s}$ where $\vec{s}$ is the unit tangent vector in S-direction.

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Let $\vec{s}, \vec{n}, \vec{b}$ be triply orthogonal unit-vectors, tangent, principal normal and abnormal to the spatial curves of congruencies and denote $\frac{\partial}{\partial s}, \frac{\partial}{\partial n}$ and $\frac{\partial}{\partial b}$ respective directional derivative along these curves. Let (k_s, k_n', k_b'') and $(\tau_s, \sigma'_n, \sigma''_b)$ be the curvatures and the torsions of the curves.

On considering $\vec{u}$ and $\vec{v}$ as $\vec{u} = u\vec{s}$ and $\vec{v} = v\vec{s}$ and by using the Frenet formulae, we can write

$$\nabla^2\vec{u} = \frac{\partial^2 u}{\partial s^2}\vec{s} + 2\frac{\partial u}{\partial s}k_s\vec{n} + uk_s\tau_s\vec{n} - u\left(k_s^2 + \sigma'^2_n + k'^2_n + \sigma^{n2}_b + k''^2_b\right)\vec{s} \quad \text{......... (2.5)}$$

Further, on using equations (2.1) and Frenet Formulae in the equations (2.1) (2.2), (2.3) and (2.4) we obtain

$$\frac{\partial(\rho u)}{\partial s} + \rho u(\theta_{ns} + \theta_{bs}) = 0 \quad \text{......... (2.6)}$$

$$u\frac{\partial u}{\partial s} = v\left[\frac{\partial^2 u}{\partial s^2} - u\left(k_s^2 + \sigma'^2_n + k'^2_n + \sigma''^2_b + k'''^2_b\right)\right] + \frac{f}{\tau}(v-u) - Du \quad \text{..........(2.7)}$$

$$\frac{\partial(\mathrm{Nv})}{\partial s} + Nv(\theta_{ns} + \theta_{bs}) = 0 \quad \text{....... (2.8)}$$

$$\tau v\frac{\partial v}{\partial s} = u - v \quad \text{......... (2.9)}$$

Where $D = \frac{\sigma B_0^2}{\rho}$

Now, the following two cases arise :-

Case I $v = v_0$ = constant

Case II $u = u_0$ = constant and $\tau \to \infty$

In the first case from equation (2.9), we find

$u = v_0$

And consequently equations (2.6), (2.7) and (2.8) yield

$$\rho = N = k_{le} - \int(\theta_{ns} + \theta_{bs})ds$$

And

$$B_0^2 = \left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right)\left(-\frac{\mu}{\sigma}\right)$$

Thus,

$$B_0 \propto \left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right)^{\frac{1}{2}} \quad \text{........... (2.10)}$$

$$\Rightarrow B_0 = i\frac{\mu}{\sigma}\left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right) \text{ where } B_0 \text{ is imaginary}$$

Thus, the phase-angles of magnetic lines are orthogonal that of the streamlines of the dusty as flow and further they are parabolic in nature

Following the same process of calculation as in the first case, we get

$B_0^2 = \left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right)\left(-\frac{\mu}{\sigma}\right)$ and $v = u_0$ which is exactly the same equations (2.10) and consequently the result of case I follow immediately.

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We study steady dusty gas flow in osculating plane. i.e. plane of tangent and normal.

Let $\vec{u} = u(\mathrm{n})\vec{\mathrm{s}}$ and $\vec{v} = v(\mathrm{n})\vec{\mathrm{s}}$ i.e. both $\vec{u}$ and $\vec{v}$ are parallel to tangential $\vec{\mathrm{s}}$ direction. From eqs. 2.1 and 2.2 ρ and N are found as in case I and II.

From eq. (2.4) we have $\tau v^2 \frac{\partial v}{\partial s} = (u-v)$ and $\tau v^2 k_s \vec{n} = (u-v)\vec{s}$ .This shows that either $(u-v)=0$ or $\tau v^2 k_s = 0$ .If $(u-v)=0$ then $\tau v^2 \frac{\partial v}{\partial s} = 0$ .This shows that either $v=0$ orv= constant.Thus either the flow does not exist or the flow is uniform. Hence the flow is uniform and in this case $k_s = 0$. Thus the flow is unidirectional parallel straight line and reduce to case 1.

Similarly after decomposing eq. (2.3) in Frenet frame field, we get u = constant and

$$u\left(\sigma'^2_n - k'_n + \sigma_b''^2 - k''^2_b\right) + \frac{f}{\tau}(v-u) = Du$$

If $\tau \to \infty$, then the flow reduces to case II.

If we consider the flow in the rectifying plane. i.e. the plane of tangent and binomial, the calculations are carried out as above and the flow analysis is similar to the cases I and II.

3. Flow of Unsteady Fluid And Dust Particles

The basic equations of motion of unsteady flow in Frenet Frame filed system with the assumptions $\bar{u} = u\bar{s}$ and $\bar{v} = v\bar{s}$ are decomposed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial s} + \rho u(\theta_{ns} + \theta_{bs}) = 0 \qquad \text{… (3.1)}$$

$$\frac{\partial \vec{u}}{\partial t} + u\frac{\partial u}{\partial s} = v\left[\frac{\partial^2 u}{\partial s^2} - u\left(k_s^2 + \sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right)\right] + \frac{f}{\tau}(v-u) - Du \qquad \text{… (3.2)}$$

$$\frac{\partial N}{\partial t} + \frac{\partial(Nv)}{\partial s} + Nv(\theta_{ns} + \theta_{bs}) = 0 \qquad \text{…(3.3)}$$

and

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = \frac{1}{\tau}(u-v) \qquad \text{… (3.4)}$$

Here, we come across the following cases :-

Case (A) $v=v_0$ (constant)
Case (B) $u=u_0$ (constant) and $\tau \to \infty$
Case (C) $u=u_0$ (constant) and v = v(t)

In case (A) from (3.4), we find

$u = v_0$

And from equations (3.3), we get

$$B_0^2 = \left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right)\left(-\frac{\mu}{\sigma}\right)$$

which is again equation (2.10) of the last article and so the results of case I of the last article are as such applicable in the case also.

In case (B) also, we get to same expression for B_0^2 together with

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = 0$$

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Which implies that v must be constant and so the discussion in this case too is exactly the same as of Case I of last article.
In the last case (C) from equations (3.2) and (3.4) we have

$$u_0 v\left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right) = \frac{f}{\tau}\left(\vec{v} - \vec{u}\right) - Du_0 \qquad \text{.......(3.5)}$$

and

$$v = u_0 + ke^{-t/r} \qquad \text{....... (3.6)}$$

where k is an arbitrary constant.

The eq. (3.5) show that the phase angles of magnetic lines need not be orthogonal to those of streamlines of dusty gas flows but they are parabolic in nature. Also eq. (3.6) shows that v is not a constant.

Now we study the flow in osculating and rectifying planes. Let $\bar{u}$ = u(n,t)$\bar{s}$ and $\bar{v}$ = u(n,t)$\bar{s}$ i.e. both $\bar{u}$ and $\bar{v}$ are parallel to tangential $\bar{s}$ direction. Using Frenet formulae, the basic equation of motion are decomposed as:

$$\frac{\partial \rho}{\partial t} + u\left[\frac{\partial \rho}{\partial s} + \rho\left(\theta_{ns} + \theta_{bs}\right)\right] = 0 \qquad \text{....... (3.7)}$$

$$\frac{\partial u}{\partial t} = v\left[\frac{\partial^2 u}{\partial s^2} + u\left(\sigma'^2_n + k'^2_n + \sigma''^2_b + k''^2_b\right)\right] + \frac{f}{\tau}\left(v - u\right) - Du \qquad \text{....... (3.8)}$$

$$u^2 k_s = -2k'_n v \frac{\partial u}{\partial n} \qquad \text{....... (3.9)}$$

$$\frac{\partial N}{\partial t} + v\left[\frac{\partial N}{\partial s} + N\left(\theta_{ns} + \theta_{bs}\right)\right] = 0 \qquad \text{....... (3.10)}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\tau}\left(u - v\right) \qquad \text{....... (3.11)}$$

and $v^2 k_s = 0$

Which implies $k_s = 0$

On using (3.11) in (3.9), we have

$$\frac{\partial u}{\partial n} = 0$$

Where immediately gives that

$$u = u(\mathrm{t})$$

On differentiating equation (3.8) with respect to ‘t’ we get

$$\frac{\partial^2 u}{\partial t^2} = v\left[\left(\sigma'^2_n - k'^2_n - \sigma''^2_b - k''^2_b\right) - D - \frac{f}{\tau} - \frac{1}{\tau}\right]\frac{\partial u}{\partial t}$$
$$+ \frac{1}{\tau}\left[-D + v\left(\sigma'^2_n - k'^2_n - \sigma''^2_b - k''^2_b\right)u\right]$$

Which can be written as

$$\frac{\partial^2 u}{\partial t^2} + A\frac{\partial u}{\partial t} + Bu = 0 \qquad \text{....... (3.12)}$$

Where

$$A = v\left[\left(\sigma'^2_n - k'^2_n - \sigma''^2_b - k''^2_b\right) - D - \frac{f}{\tau} - \frac{1}{\tau}\right] \qquad \text{....... (3.13)}$$

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$$B=\frac{1}{\tau}\left[-D+v\left(\sigma'^2_n-k'^2_n-\sigma''^2_b-k''^2_b\right)\right] \qquad \text{……. (3.14)}$$

Differentiating eq. (3.11), w.r.t. t and simplifying, we obtain

$$\frac{d^2v}{dt^2}+A\frac{dv}{dt}+Bv=0 \qquad \text{……. (3.15)}$$

Where, A and B are given by eqs (3.13) and (3.14) respectively.

Thus, u and v satisfy the same second order differential equation

$$D^2+AD+B=0 \quad \text{where} \quad D=\frac{d}{dt}$$

Hence, either u=v or u is parallel to v(u v) and they are given by

$$u=c_1e^{\left[\frac{-A+\sqrt{A^2-4B}}{2}\right]t}+c_2e^{\left[\frac{-A-\sqrt{A^2-4B}}{2}\right]} \qquad \text{……. (3.16)}$$

and

$$u=d_1e^{\left[\frac{-A+\sqrt{A^2-4B}}{2}\right]t}+d_2e^{\left[\frac{-A-\sqrt{A^2-4B}}{2}\right]} \qquad \text{……. (3.17)}$$

where c_1, c_2, d_1 and d_1 are arbitrary constants

The analysis of flow in all the cases discussed above in rectifying plane can also be made similarly.
In what follows, we shall study propagation of shock waves in the atmosphere of a rotating star.

4. Basic Equations For Shock Propagation

The fundamental equations governing the motion of gravitating and rotating atmosphere in spherical symmetry, are :

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+u\frac{\partial u}{\partial r}+\frac{2u}{r}=0, \qquad \text{……. (4.1)}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+\frac{1}{p}\frac{\partial p}{\partial r}+\frac{g_0r_0}{r^2}-\frac{v^2}{r}=0, \qquad \text{……. (4.2)}$$

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)(v.r)=0, \qquad \text{……. (4.3)}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}-a^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0, \qquad \text{……. (4.4)}$$

Where r is the radial coordinate, u,v are the radial and azimuthally components of the particle velocity, p, ρ and a are the pressure, density and speed of sound, and r_0 and g_0 are the reference distance the acceleration due to gravity at r_0.

The shock conditions may be written as,

$$\frac{p'}{p}=\frac{1}{\beta} \qquad \text{……. (4.5)}$$

$$\frac{u'}{a}=(1-\beta)\sqrt{\frac{2}{(\gamma+1)\beta-(\gamma-1)}}+M=\sqrt{\frac{2(1-\beta)^2}{(\gamma+1)\beta-(\gamma-1)}}+M=\zeta+M \qquad \text{…. (4.6)}$$

$$\frac{\rho'}{\rho}=\frac{(\gamma+1)-\beta(\gamma-1)}{(\gamma+1)\beta-(\gamma-1)}=\phi \qquad \text{……. (4.7)}$$

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$$\frac{a'}{a} = (\phi\beta)^{1/2} = \psi, \qquad \ldots\ldots (4.8)$$

Where γ is the ration of specific heats of the gas, and $M = \frac{u}{a}$, and mach number of the flow in front of the shock. The primed variables are used to indicate values behind the shock, ϕ, ψ and ζ are functions of β.

Supposing the flow in front of the shock to be steady, the continuity, equation, momentum equation and the equation of rotation.

i.e. equations (4.1-4.3) may be written as

$$u\frac{d\rho}{dr} + \rho\frac{du}{dr} + 2\frac{\rho u}{r} = 0 \qquad \ldots\ldots (4.9)$$

$$u\frac{du}{dr} + \frac{1}{\rho}\frac{dp}{dr} + \frac{g_0 r_0}{r^2} - \frac{v_0^2}{r} = 0 \qquad \ldots\ldots (4.10)$$

$$u\frac{d}{dr}(\mathrm{v.r}) = 0 \quad \text{i.e.} \quad v.r = v_0 r_0 \qquad \ldots\ldots (4.11)$$

Combining the equation of state with relation for the speed of sound, we may write

$$\frac{p}{\rho} = \frac{RT}{m} = \frac{a^2}{\gamma}, \qquad \ldots\ldots (4.12)$$

Where T is the temperature, R is the universal gas constant and m is the

Molecular weight of the gas

From (2.12), it is easy to prove

$$\left(\frac{1}{\rho}\frac{d\rho}{dr} - \frac{1}{p}\frac{dp}{dr}\right) = \frac{2}{a}\frac{da}{dr} = \left(\frac{1}{T}\frac{dT}{dr} - \frac{1}{m}\frac{dm}{dr}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

Using (4.9) and (4.10), we have

$$\left(1-\gamma M^2\right)\frac{1}{u}\frac{du}{dr} = \left(\frac{1}{p}\frac{dp}{dr} - \frac{1}{\rho}\frac{d\rho}{dr}\right) - \frac{2}{r} + \frac{g_0 r_0 \rho}{r^2 p} - \frac{v_0^2 \rho}{rp} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4.14)$$

The use of these equations implicitly assumes that

1. The flow is essentially one dimensional.
2. The gas is perfect as far as the shock wave relation are concerned.
3. If shock is propagating in an ionized gas, any magnetic field lies in the direction of the motion of the shock,
4. Radiation effect are not taken into account.
5. A continuous approach is valid.

The derivation of the equations that accounts for the generation and propagation of disturbances on the shock from follows the analysis of Whitham [20].

Applying the rule described by the Whitham [20] the characteristic relation which applied immediately behind the shock may be written as:

$$\frac{dp'}{dr}+a'\rho\frac{du'}{dr}+\frac{a'\rho'}{u'+a'}\left(\frac{g_0r_0}{r^2}+\frac{2a'u'}{r}-\frac{v'^2}{r}\right)=0, \qquad \text{………………. (4.15)}$$

Substituting (4.5)-(4.8) in the above equation we may obtain :

$$\frac{d\beta}{dr}=\frac{\dfrac{\left\{\beta(\gamma+1)-(\gamma-1)\right\}^2\zeta\beta}{4\gamma\beta\zeta+\psi\gamma(1-\beta)\left\{\gamma(\beta-1)+(\beta+3)\right\}}}{\left(1-\gamma M^2\right)}\times$$

$$\left[\left(\frac{1}{T}\frac{dT}{dr}-\frac{1}{M}\frac{dm}{dr}\right)\left\{\gamma M^2\left(\frac{\psi}{M\beta}-\frac{\zeta\psi\gamma}{2\beta}-\frac{\phi\beta}{\beta}\right)+\frac{\zeta\psi\gamma}{2\beta}\right\}+\left(\frac{g_0r_0m}{RTr^2}\left(\zeta+M+\psi\right)^{-1}\right)\right.$$

$$\left\{\frac{\psi^2\gamma(\zeta+M)\left(1-\gamma M^2\right)}{\beta}-\frac{\psi\gamma M}{\beta}\left(\zeta+M+\psi\right)+\frac{\gamma M^2\phi\beta\left(\zeta+M+\psi\right)}{\beta}\right\}$$

$$\left.-\left\{\left(\frac{\psi\gamma M-\phi\beta}{\beta}\right)\left(\zeta+M\psi\right)+\frac{\psi}{\beta}\left(1-\gamma M^2\right)\right\}\left(\zeta+M+\psi\right)^{-1}\frac{r_0^2\omega_0^2m}{RTr}\right], \qquad \text{………. (4.16)}$$

Where $v'=v_0=r_0\omega_0,\omega_0$, w_0 being the angular rotation of the atmosphere.

In equation (4.16) the change of the density, shock strength with distance r is function of temperature, molecular weight, Mach number distribution in front of the shock, force of gravitation and rotation of the atmosphere.

In case of steady atmosphere, the Mach number distribution is given by

$$\frac{1}{M}\frac{dM}{dr}=\frac{1}{\left(1-\gamma M^2\right)}\left[\frac{\left(1+\gamma M^2\right)}{2}\left(\frac{1}{T}\frac{dT}{dr}-\frac{1}{m}\frac{dm}{dr}\right)+\frac{mg_0r_0}{RTr^2}-\frac{2}{r}-\frac{\omega_0^2r_0^2m}{RTr}\right], \text{…. (4.17)}$$

The apparent singularity is equation (4.16) and (4.17 at $M^2=\frac{1}{\gamma}$ does not necessarily have any physical significance as far as the behavior of M and β is concerned.

The temperature distribution in equations (4.16) and (4.17) must be determined by energy consideration. A suitable form of energy equation is :

$$\rho\,\mathrm{u}\frac{d}{dx}\left(C_pT+\frac{1}{2}u^2\right)=\rho q-\frac{d}{dx}\left[K\frac{dT}{dx}\right]-\rho ug, \qquad \text{………… (4.18)}$$

where K is the thermal conductivity, C_p is the specific heat at constant pressure, q is the time rate of the heat addition per unit mass from outside due to thermal radiation or heating by steady shock waves (q is negative from radioactive losses).

In the special case of non-heat conducting gas, the energy equation reduces to

$$\frac{M^2}{u}\frac{du}{dr}+\frac{2}{(\gamma-1)}\frac{1}{a}\frac{da}{dr}+\frac{g_0r_0}{a^2r^2}-\frac{\omega_0^2r_0^2}{a^2r}=0 \qquad \text{……….. (4.19)}$$

Combining (4.9), (4.10), (4.12) and (4.19), we obtain

$$\frac{1}{T}\frac{dT}{dr}-\frac{1}{m}\frac{dm}{dr}=\frac{(\gamma-1)}{(1-M^2)}\left[\frac{2M^2}{r}-\frac{mg_0r_0}{\gamma RTr^2}+\frac{\omega_0^2r_0^2m}{rRT\gamma}\right] \quad \text{……….} (4.20)$$

5. Discussion:-

It is not easy to draw general conclusion from equation (4.16) and thus the solution of the problem may be discussed from the simplified form of the resulting equation for the limiting cases of very strong and very weak shock waves along with the differential astrophysical situations.

Writing $\frac{1}{\beta}=1+\alpha$, where $\alpha>>1$ for strong shock and $\alpha<<1$ for weak shock, in the case of strong shock.

$$\zeta=\left[\frac{2\alpha^2}{(\gamma+1)(1+\alpha)-(\gamma-1)(1+\alpha^2)}\right]^{1/2}=\mu, \quad \text{........} (5.1)$$

$$\phi=\frac{2+\alpha(\gamma+1)}{2-\alpha(\gamma-1)}=\delta, \quad \text{.........} (5.2)$$

$$\psi=\left[\left(\frac{2+\alpha(\gamma+1)}{2-\alpha(\gamma-1)}\right)\frac{1}{(1+\alpha)}\right]^{1/2}=v, \quad \text{.........} (5.3)$$

and

$$\frac{d\alpha}{dr}=\frac{\{(1+\alpha)\mu(\gamma-1)-(\gamma+1)\}}{(1-\gamma M^2)\{4\gamma\mu(1+\alpha)+\alpha\gamma v(4+\alpha(3-\gamma))\}}\times$$
$$\left[\left(\frac{1}{T}\frac{dT}{dr}\right)\left\{\gamma M^2\left(\frac{v}{M}-\frac{\mu v}{2}-\delta(1+\alpha)\right)+\frac{\gamma v\gamma}{2}\right\}+\frac{g_0r_0m_0}{r^2RT}(\mu+v+M)^{-1}\right.$$
$$\{v\gamma M(\mu+v+M)-\delta(1+\alpha)(\mu+v+M)+v(1-\gamma M^2)\}$$
$$+\frac{2}{r}(\mu+v+M)^{-1}\{v^2\gamma(\mu+M)(1-\gamma M^2)-v\gamma M(\mu+v+M)+\gamma M^2(1+\alpha)\delta(\mu+v+M)\}$$

$$\left.-\frac{r_0^2\omega_0^2m}{rRT}\{(v\gamma M-\delta(1+\alpha))(\mu+v+M)+v(1-\gamma M^2)\}(\mu+v+M)^{-1}\right] \quad \text{....} (5.4)$$

When atmosphere is static infront of the shock, M=0 and so

$$\frac{d\alpha}{dr}=A\left[\frac{1}{T}\frac{dT}{dr}\frac{\mu v\gamma}{2}+\frac{g_0r_0m_0}{r^2RT}\left\{\frac{v-\delta(1+\alpha)(\mu+v)}{\mu+v}\right\}+\frac{2}{r}\frac{\mu\gamma v^2}{(\mu+v)}\right.$$
$$\left.-\frac{r_0^2\omega_0^2m}{rRT}\left\{\frac{v+\delta(1+\alpha)(\mu+v)}{\mu+v}\right\}\right], \quad \text{……} (5.5)$$

where,

$$\mathrm{A}=\frac{(1+\alpha)\mu(\gamma-1)-(\gamma+1)}{4\gamma\mu(1+\alpha)+\alpha\gamma v\{4+\alpha(3-\gamma)\}} \quad \text{…….} (5.6)$$

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In the case of weak shock, for which $\alpha << 1$, the equation (5.4) takes the form,

$$\frac{d\alpha}{dr}=\frac{\mu_1(\gamma-1)+(\gamma+1)}{(1-\gamma M^2)4\gamma(\mu_1+\alpha)}\left[\frac{1}{T}\frac{dT}{dr}\left\{\gamma M^2\left(\frac{v_1}{M}-\frac{\mu_1 v_1}{2}-\delta_1(1+\alpha)\right)+\frac{\mu_1 v_1\gamma}{2}\right\}\right.$$
$$+\frac{g_0 r_0 m_0}{r^2 RT}(\mu_1+v_1 M)^{-1}\left\{v_1\gamma M(\mu_1+v_1+M)-\delta_1(1+\alpha)(\mu_1+v_1+M)+\mathrm{v}_1(1-\gamma M^2)\right\}$$
$$+\frac{2}{r}(\mu_1+v_1 M)^{-1}\{(v_1^2\gamma(\mu_1+M)(1-\gamma M^2)-2\mathrm{v}_1\gamma M(\mu_1+v_1+M)+\gamma M^2(1+\alpha)\delta_1(\mu_1+v_1+M)\}$$
$$\left.-\frac{r_0^2\omega_0^2 m}{rRT}\{(v_1\gamma M-\delta_1(1+\alpha)(\mu_1+v_1 M)+v_1(1-\gamma M^2)(\mu_1+v_1+M)^{-1}\right]$$

….. (5.7)

where,

$$\mu_1=\sqrt{\frac{2}{\gamma-1}}\left\{1+\frac{1}{2}\left(\frac{\gamma-1}{\gamma+2}\right)\right\}\alpha \quad ……(5.8)$$

$$\delta_1=1+\alpha\gamma \quad ……(5.9)$$

$$v_1=\left\{1+\frac{\alpha(\gamma-1)}{2}\right\} \quad ……(5.10)$$

Now, we write

$$\mu_1+v_1=\omega_1=\sqrt{\frac{2}{\gamma+1}}\left\{1+\frac{1}{2}\left(\frac{\gamma-1}{\gamma+1}\right)\right\}\alpha+\left(1+\frac{\alpha(\gamma-1)}{2}\right), \quad …… (5.11)$$

$$\mu_1 v_1=\omega_2=\sqrt{\frac{2}{\gamma+1}}\left(1+\frac{\alpha(\gamma-1)(\gamma+3)}{2(\gamma+1)}\right), \quad …… (5.12)$$

$$\mathrm{v}_1^2=1+\alpha(\gamma-1), \quad ……. (5.13)$$

$$\delta_1(1+\alpha)=1+\alpha(\gamma-1), \quad ……. (5.14)$$

and

$$(1+\alpha)\mu_1=\mu_1 \quad ……. (5.15$$

Substituting the values from (5.8) – (5.15), in equation (5.7), we have

$$\frac{d\alpha}{dr}=\left\{\frac{\dfrac{\mu_1(\gamma-1)-(\gamma+1)}{4\gamma(\mu_1+\alpha)}}{1-\gamma M^2}\right\}\left[\left(\frac{1}{T}\frac{dT}{dr}\right)\left\{\gamma M^2\left(\frac{v_1}{M}-\frac{\omega_2}{2}-(1+\alpha(\gamma+1))\right)+\frac{\omega_2\gamma}{2}\right\}\right.$$
$$+(\omega_1+M)^{-1}\left\{\frac{g_0 r_0 m_0}{r^2 RT}\left\{\gamma v_1 M(\omega_1+M)-(1+\alpha(\gamma+1))(\omega_1+M)\right\}\right.$$
$$+\frac{2}{r}\{(1+\alpha(\gamma-1)\gamma(\mu_1+M)(1-\gamma M^2)-2v_1\gamma M(\omega_1+M)$$

$$+\gamma M^2\left(1+\alpha\left(\gamma+1\right)\right)\left(\omega_1+M\right)\Big\}-\frac{r_0^2\omega_0^2 m}{rRT}\left(v_1\gamma M-\left(1+\alpha\left(\gamma+1\right)\right)\left(\omega_1 M+v\left(1-\gamma M^2\right)\right)\right)\Big\}\Big]$$
…… (5.16)

Using the condition of static atmosphere, we have from (5.16)

$$\frac{d\alpha}{dr}=\frac{\mu_1(\gamma-1)-(\gamma+1)}{4\gamma(\mu_1+\alpha)}\left[\frac{1}{T}\frac{dT}{dr}\left(\frac{\gamma\omega_2}{2}\right)+\left\{-\frac{g_0r_0m_0}{r^2RT}\left(1+\alpha\left(\gamma+1\right)\right)\right.\right.$$
$$\left.\left.+\frac{2}{\gamma}\left(1+\alpha\left(\gamma-1\right)\right)\frac{\gamma\mu_1}{\omega_1}-\frac{r_0^2\omega_0^2 m}{rRT}\left(\left(1+\alpha\left(\gamma+1\right)\right)+\frac{v_1}{\omega_1}\right)\right\}\right] \quad \text{…… (5.17)}$$

Or

$$\frac{d\alpha}{dr}=\frac{\mu_1(\gamma-1)-(\gamma+1)}{4\gamma(\mu_1+\alpha)}\left[\frac{1}{T}\frac{dT}{dr}\frac{\gamma\omega_2}{2}-\frac{g_0r_0m_0}{r^2RT}\left(1+\alpha\left(\gamma+1\right)\right)\right.$$
$$\left.+\frac{2}{\gamma}\left(1+\alpha\left(\gamma-1\right)\right)\frac{\gamma\mu_1}{\omega_1}-\frac{r_0^2\omega_0^2 m}{rRT}\left(\left(1+\alpha\left(\gamma+1\right)\right)+\frac{v_1}{\omega_1}\right)\right] \quad \text{……. (5.18)}$$

For an isothermal atmosphere, $\frac{dT}{dr}$ =0. In such a atmosphere, the effect of gravity and rotation of the fluid on the outward moving shock is to cause decrease in strength with distance from the origin while the effect of the spherical geometry of the motion is to cause an increase in strength, Inward moving shock behaves in opposite manner.
In case of very weak shock in which M and α, both are small and M is large in comparison of α, we have-

$$\frac{d\alpha}{dr}=\frac{\gamma+1}{4\alpha\gamma}\left[-\frac{1}{T}\frac{dT}{dr}\left(M+\frac{1}{2}\sqrt{\frac{2}{\gamma+1}}\right)\gamma-\frac{g_0r_0m_0}{r^2RT}\left(\gamma M-1\right)\right)+\frac{2\gamma M}{r}$$
$$\left.-\frac{r_0^2\omega_0^2 m}{rRT}\left(\gamma-1\right)M\right] \quad \text{…… (5.19)}$$

In stationary atmosphere,

$$\frac{d\alpha}{dr}=\frac{\gamma+1}{4\alpha\gamma}\left[-\frac{1}{T}\frac{dT}{dr}\frac{\gamma}{2}\sqrt{\frac{2}{\gamma+1}}+\frac{g_0r_0m_0}{r^2RT}\right] \quad \text{….. (5.20)}$$

The critical temperature gradient is given by

$$\frac{dT}{dr}=\frac{2}{\gamma}\sqrt{\frac{\gamma+1}{2}}\left(\frac{g_0r_0m_0}{r^2R}\right) \quad \text{….. (5.21)}$$

The density strength remains constant if,

$$\frac{1}{T}\frac{dT}{dr}=\frac{\left(\zeta+M+\psi\right)^{-1}}{\frac{\gamma M^2}{\beta}\left(\frac{\psi}{M}-\frac{\zeta\psi\gamma}{2}-\phi\beta\right)+\frac{\zeta\psi\gamma}{2\beta}}$$
$$\left[\frac{r_0^2\omega_0^2 m}{rRT}\left\{\frac{\psi\gamma M-\phi\beta}{\beta}\left(\zeta+M+\psi\right)+\frac{\psi}{\beta}\left(1-\gamma M^2\right)\right\}\right.$$
$$-\frac{g_0r_0m_0}{r^2RT}\left\{\frac{\psi\gamma M\left(\zeta+M+\psi\right)}{\beta}-\frac{\phi\beta\left(\zeta+M+\psi\right)}{\beta}+\frac{\psi}{\beta}\left(1-\gamma M^2\right)\right\}$$

$$-\frac{2}{r}\left\{\frac{\psi^{2}\gamma(\zeta+M)}{\beta}\left(1-\gamma M^{2}\right)-\frac{\psi\gamma M(\zeta+M+\psi)}{\beta}+\frac{\gamma M^{2}\phi\beta(\zeta+M+\psi)}{\beta}\right\}\Bigg]$$

…… (5.22)

This equation may be substituted into (4.17) to obtain Mach number gradient as given by-

$$\frac{1}{M}\frac{dM}{dr}=\frac{\begin{gathered}\frac{1+\gamma M^{2}}{2}\left[\frac{r_0^2\omega_0^2 m}{rRT}\left\{\frac{\psi\gamma M-\phi\beta}{\beta}(\zeta+M+\psi)+\frac{\psi}{\beta}\left(1-\gamma M^{2}\right)\right\}\right.\\ -\frac{g_0 r_0 m_0}{r^2RT}\left\{\frac{\psi\gamma M(\zeta+M+\psi)}{\beta}-\frac{\phi\beta(\zeta+M+\psi)}{\beta}+\frac{\psi}{\beta}\left(1-\gamma M^{2}\right)\right\}\\ -\frac{2}{r}\left\{\frac{\psi^{2}\gamma(\zeta+M)}{\beta}\left(1-\gamma M^{2}\right)-\frac{\psi\gamma M(\zeta+M+\psi)}{\beta}+\frac{\gamma M^{2}\phi\beta(\zeta+M+\psi)}{\beta}\right\}\end{gathered}}{\left(1-\gamma M^{2}\right)\left[(\zeta+M+\psi)\left\{\frac{\gamma M^{2}}{\beta}\left(\frac{\psi}{\beta}-\frac{\zeta\psi\gamma}{2}-\phi\beta\right)+\frac{\zeta\psi\gamma}{2\beta}\right\}+\right.}$$

$$\frac{1}{\left(1-\gamma M^{2}\right)}\left(-\frac{1}{M}\frac{dm}{dr}+\frac{m_0 g_0 r_0}{r^2RT}-\frac{2}{r}-\frac{\omega_0^2 r_0^2 m}{rRT}\right)\Bigg]$$

This simple analytical result obtained in above cases is relevant to most of the problems involving the propagation of shock waves in a gravitation atmosphere. This approximate theory is expected to give good results in those cases where the rate of increase in the shock strength is extremely large. The relative effect due to gravitation, rotation and due to area change of the motion can be obtained from the above results, which effects predominated depends on the relative magnitude of the term in above result.

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