Research Paper
Impact Factor: 3.567
Peer Reviewed Journal

IJMDRR
E- ISSN –2395-1885
ISSN -2395-1877

FUZZY GAME MATRIX SOLUTIONS USING AVERAGE WEIGHTED APPROACH

Dr. M. S. Annie Christi* N. Kalpana**
**Associate Professor, Department of Mathematics,Providence College for Women, Coonoor, India.*
***Department of Mathematics, Providence College for Women, Coonoor, India.*

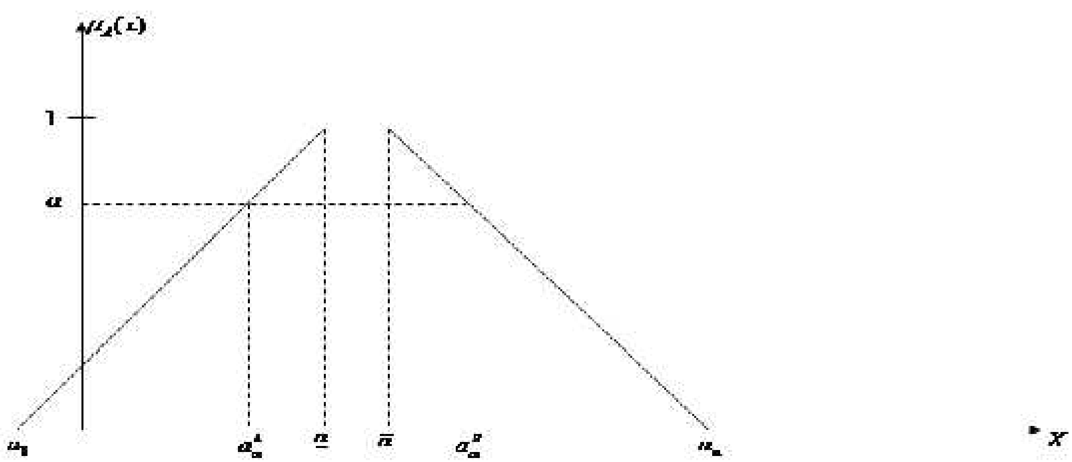
Abstract
Two person zero- sum game with trapezoidal fuzzy numbers are considered. An interval programming problem is applied for each player so as to transform into bi-objective linear problem using interval inequality relations. Applying the weighted average method, we get an improvement over the results of optimal pay-off value of the game.

Keywords: Interval programming problem, Bi-objective linear problem, trapezoidal fuzzy number, weighted average method.

1.1. Definition: **Definition: (Trapezoidal fuzzy number) (Tr FN)**:
A fuzzy number A is called a trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} 0 & , x < a_l, x > a_u, \\ \dfrac{x - a_1}{\underline{a} - a_1} & , a_l \le x \le \underline{a}, \\ 1 & , \underline{a} \le x \le \overline{a}, \\ \dfrac{a_u - x}{a_u - \overline{a}} & , \overline{a} < x \le a_u. \end{cases}$$

The TrFN A is denoted by the quadruplet A= $(a_1, \underline{a}, \overline{a}, a_u)$ and has the shape of a trapezoid.



Further the α-cut of the TrFN A= $(a_1, \underline{a}, \overline{a}, a_u)$ is the closed interval

$$A_\alpha = [a_\alpha^L, a_\alpha^R] = [(\underline{a} - a_1)\alpha + a_1, -(a_u - \overline{a})\alpha + a_u], \ \alpha \in (0,1].$$

Next let A= $(a_l, \underline{a}, \overline{a}, a_u)$ and B= $(b_l, \underline{b}, \overline{b}, b_u)$ be two TrFN, then using the ~ -cuts one can compute A*B where * may be (+), (-), (.), $\vee$ or $\wedge$ operation. In this context it can be verified that

$$(i) A(+)B = (a_l + b_l, \underline{a} + \underline{b}, \overline{a} + \overline{b}, a_u + b_u),$$
$$(ii) - A = (-a_u, -\overline{a}, -\underline{a}, -a_l),$$
$$(iii) A - B = (a_l - b_u, \underline{a} - \overline{b}, \overline{a} - \underline{b}, a_u - b_l)$$
$$(iv) kA = (ka_l, k\underline{a}, k\overline{a}, ka_u), k > 0,$$

1.2. Definition: (TFN)

A TFN $\ddot{a}$ = ($\dot{a}$, a, $\acute{a}$) is a special type of fuzzy set on the set $\Re$ of real numbers, whose membership function is defined as follows

$$\mu_{\ddot{a}}(x) = \begin{cases} \frac{x - \dot{a}}{a - \dot{a}}, & \dot{a} \le x \le a \\ \frac{\acute{a} - x}{\acute{a} - a}, & a < x \le \acute{a} \end{cases}$$

where a is the mean value of $\ddot{a}$, $\dot{a}$ and $\acute{a}$ represents the left and right spread respectively. Note that if $\dot{a}$ = $\acute{a}$ = a, then $\ddot{a}$ is reduced to a crisp number. The set of all TFNs are denoted by $\ddot{\mathcal{F}}(\Re)$.

1.3 Definition: (α- cut sets):

A α-cut set of a TFN, $\ddot{a}$= ($\dot{a}$, a, $\acute{a}$) is a crisp subset of $\Re$, which is defined as $\ddot{a}_\alpha$= {x: $\mu_{\ddot{a}}$ (x) ≥α} where 0 ≤α≤1. Using the membership function $\mu_{\ddot{a}}$(x) it can be easily proved that $\ddot{a}_\alpha$ is a closed interval and $\ddot{a}_\alpha$ = [$\dot{a}$+ α (a -$\dot{a}$), $\acute{a}$-α ($\acute{a}$-a)] = [a_L^α, a_R^α]. Where a_L^α = $\dot{a}$+α (a-$\dot{a}$) and a_R^α =$\acute{a}$-α ($\acute{a}$-a). The set of all α-cut values of TFNs is denoted by $\ddot{\mathcal{F}}_\alpha(\Re)$.

2. Basic Interval Arithmetic

2.1. Definition: ([16]):

An interval number is defined as $\ddot{a}$= [a_L ,a_R] = {x$\in \Re : a_L$≤x≤a_R},$\Re$ is the set of all real numbers. The numbers a_L, a_R are called respectively the lower and upper limits of the interval $\ddot{a}$. An interval number $\ddot{a}$ can also be represented in mean-width form as $\ddot{a}$ = ⟨m(a), w(a)⟩, where m (a) =$\frac{1}{2}$(a_L+a_R) and w(a) = $\frac{1}{2}$ (a_R-a_L) are the mid-point and half-width of the interval $\ddot{a}$. The set of all interval numbers in $\Re$ is denoted by I ($\Re$).The basic interval arithmetic are given as follows. Let

$\ddot{a}$ = [a_L, a_R] and $\ddot{b}$= [b_L, b_R] be two interval numbers

Then

$\ddot{a}$+$\ddot{b}$ = [$a_L + b_L, a_R + b_R$], $\ddot{a}$-$\ddot{b}$ = [$a_L - b_R, a_R - b_L$] and

$$\lambda\ddot{a} = \begin{cases} [\lambda a_L, \lambda a_R] & if\ \lambda \ge 0 \\ [\lambda a_R, \lambda a_L] & if\ \lambda < 0 \end{cases}$$

where λ is a real scalar.

2.2. Definition: ([18])

The crisp equivalent forms of interval inequality constraints $\ddot{a}z \le_I \ddot{b}$ and $\hat{a}z \ge_I \ddot{b}$ are defined as

$$\hat{a}z \leq_I \hat{b} \Rightarrow \begin{cases} a_R z \leq b_R \\ \frac{m(\hat{a}z)-m(b)}{w(\hat{a}z+w(b)} \leq \beta \end{cases}$$ and

$$\hat{a}z \leq_I \hat{b} \Rightarrow \begin{cases} a_L z \leq b_L \\ \frac{m(b)-m(\hat{a}z)}{w(\hat{a}z+w(b)} \leq \beta \end{cases} \quad (1)$$

Where $\leq_I$ and $\geq_I$ denote the interval number inequalities and $\beta \in [0,1]$ represents the minimal degree of the inequality constraints.

2.3. Definition: ([7])

Let $\hat{a} = [a_L, a_R]$ be an interval. The maximization and minimization problem with the interval valued objective function are described as follows

Max $\{\hat{a}/\hat{a} \in \Omega_1\}$ and min $\{\hat{a}/\hat{a} \in \Omega_2\}$

Which are equivalent to the following bi-objective mathematical programming problems:

Max $\{a_L, m(\hat{a})/\hat{a} \in \Omega_1\}$ and min$\{a_R, m(\hat{a})/\hat{a} \in \Omega_2\}$

Hear Ω_1 and Ω_2 are the set of constraints.

3. Mathematical Model of a Matrix Game:

Let $i \in \{1,2,\ldots\ldots m\}$ be a pure strategy available for player I and $j \in \{1,2,\ldots\ldots.n\}$ be a pure strategy available for player II. When player I chooses a pure strategy i and the player II chooses a pure strategy j, then a_{ij} is the payoff for player I and $-a_{ij}$ be a payoff for player II. The two-person zero-sum matrix game G can be represented as a pay-off matrix A = $(a_{ij})_{m\times n}$

3.1 Mixed Strategy:

Consider the game G with no saddle point, i.e. $\max_i\{min_j\ a_{ij}\} \neq min_j\{\max_i\ a_{ij}\}$. Neumann [18] introduced the concept of mixed strategy. We denote the sets of all mixed strategies, called strategy spaces, available for the players I, II by

S_I = {x= $(x_1, x_2, \ldots\ldots\ldots\ldots x_m) \in \Re_+^m$: $x_i \geq 0$; i=1, 2,……m and $\sum_{i=1}^m x_i$=1}

S_{II} = {y= $(y_1, y_2, \ldots\ldots\ldots\ldots y_m) \in \Re_+^n$: $y_j \geq 0$; j=1, 2,……n and $\sum_{j=1}^n y_i$=1}

where $\Re_+^m$ denotes the m- dimensional non-negative Euclidean space. Thus by a crisp two-person zero-sum matrix game G we mean the triplet G = (S_I, S_{II}, A).

3.2 Matrix Game with Fuzzy Pay-offs

Consider S_I, S_{II} be the strategy spaces for player I and player II respectively and $\tilde{A}$ = ($\tilde{a}_{ij}$) be the pay-off matrix where each $\tilde{a}_{ij}$= ($\underline{a}_{ij}, a_{ij}, \bar{a}_{ij}$) is a TFN. Then a two-person zero-sum matrix game with fuzzy pay-offs is the triplet ($S_I, S_{II}, \tilde{A}$).

3.1. Definition: ([3])

Let $\tilde{v}$ and $\tilde{w}$ be two TFNs. If there exists $x^* \in S_I$ and $y^* \in S_{II}$, satisfying $x^{*^T} \tilde{A}$ y$\gtrsim\tilde{v}$ $\forall y \in S_{II}$ and $x^T \tilde{A} y^*$ is a reasonable solution of $\tilde{F}\tilde{G}$ then $\tilde{v}$ (respectively,$\tilde{w}$) is called reasonable value of the player I (respectively, player II).

IJMDRR
E- ISSN –2395-1885
ISSN -2395-1877

3.2. Definition: ([3])

Let $\tilde{V}$ and $\tilde{W}$ denote the set of all reasonable values $\tilde{v}, \tilde{w}$ for players I and II respectively. Assume that there exist $\tilde{v}^* \in \tilde{V}$ and $\tilde{w}^* \in \tilde{W}$. If there does not exist any $\tilde{v} \in \tilde{V}(\tilde{v} \neq \tilde{v}^*)$ and $\tilde{w} \in \tilde{W}$ $(\tilde{w} \neq \tilde{w}^*)$ such that the following conditions $\tilde{v}^* \precsim \tilde{v}$, and $\tilde{w}^* \succsim \tilde{w}$ then $(x^*, y^*, \tilde{v}^*, \tilde{w}^*)$ is an optimal solution of the $\widetilde{FG}$. x^* (respectively, y^*) is called an optimal strategy for player I (respectively, player II) and $\tilde{v}^*$(respectively, $\tilde{w}^*$) is the value of the game $\widetilde{FG}$ for player I (respectively, player II).

According to the definitions, the optimal strategies $x^* \in S_I$ for player I and $y^* \in S_{II}$ for player II can be solving the following fuzzy mathematical problems:

Max $\{\tilde{v}\}$

Subject to $\begin{cases} x^T \tilde{A} y \succsim \tilde{v} \ \ \forall y \in S_{II} \\ x \in S_I \\ \tilde{v} \in \tilde{F}(\Re) \end{cases}$

min $\{\tilde{w}\}$

and subject to $\begin{cases} x^T \tilde{A} y \precsim \tilde{w} \ \ \forall x \in S_I \\ y \in S_{II} \\ \tilde{w} \in \tilde{F}(\Re) \end{cases}$

Since S_I and S_{II} are convex it is sufficient to consider only the extreme points (i.e. pure strategies) of and S_I and S_{II} .The following two fuzzy mathematical programming (FMP) models as

Max $\{\tilde{v}\}$

Subject to $\begin{cases} \sum_{i=1}^{m} \tilde{a}_{ij} x_i \succsim \tilde{v}(j = 1,2, \dots, n) \\ \sum_{i=1}^{m} x_i = 1 \\ x_i \geq 0 (i = 1,2, \dots \dots, m) \end{cases}$

min $\{\tilde{w}\}$

and Subject to $\begin{cases} \sum_{j=1}^{n} \tilde{a}_{ij} y_j \succsim \tilde{w}(i = 1,2, \dots, n) \\ \sum_{j=1}^{n} y_i = 1 \\ y_j \geq 0 (j = 1,2, \dots \dots, m) \end{cases}$

respectively.

The above two FMPs, can be transformed into the following interval mathematical programming models as

Max $\{\tilde{v}\}$

Subject to $\begin{cases} \sum_{i=1}^{m} (\tilde{a}_{ij} x_i)_\alpha \geq_I \tilde{v}_\alpha (j = 1,2, \dots, n) \\ \sum_{i=1}^{m} x_i = 1 \\ x_i \geq 0 (i = 1,2, \dots \dots, m) \end{cases}$

Min $\{\tilde{w}\}$

Subject to $\begin{cases} \sum_{j=1}^{n} (\tilde{a}_{ij} y_j)_\alpha \geq_I \tilde{w}_\alpha (i = 1,2, \dots, m) \\ \sum_{j=1}^{n} y_i = 1 \\ y_j \geq 0 (j = 1,2, \dots \dots, n) \end{cases}$

respectively.

Let us consider the α-cut sets of $\tilde{v}, \tilde{w}$ and $\tilde{a}_{ij}$ as

$\tilde{v}_\alpha = [\, v_L^\alpha, v_R^\alpha \,]$, $\tilde{w}_\alpha = [\, w_L^\alpha \, , w_R^\alpha \,]$ and $(\tilde{a}_{ij}) = [a_{ij}^\alpha L, a_{ij}^\alpha R]$.

Where $[v_L^\alpha, v_R^\alpha]$ is an interval and v_L^α, v_R^α denote lower and upper limits respectively. The two models can be written as

Max $\{[v_L^\alpha, v_R^\alpha]\}$

Subject to $\begin{cases} \sum_{i=1}^{m}[a_{ijL}^{\alpha}, a_{ijR}^{\alpha}]x_i \geq_I [v_L^{\alpha}, v_R^{\alpha}], \ (j = 1, 2, \ldots . n) \\ \sum_{i=1}^{m} x_i = 1 \\ x_i \geq 0 (i = 1, 2, \ldots \ldots , m) \end{cases}$

Min $\{[w_L^{\alpha}, w_R^{\alpha}]\}$

Subject to $\begin{cases} \sum_{j=1}^{n}[a_{ijL}^{\alpha}, a_{ijR}^{\alpha}]y_j \geq_I [w_L^{\alpha}, w_R^{\alpha}], \ (i = 1, 2, \ldots . m) \\ \sum_{j=1}^{n} y_i = 1 \\ y_j \geq 0 (j = 1, 2, \ldots \ldots , n) \end{cases}$

respectively.

The above two problems can be converted into the two bi-objective mathematical programming models as follows

Max $\left\{v_L^{\alpha}, \frac{v_L^{\alpha} + v_R^{\alpha}}{2}\right\}$

Subject to $\sum_{i=1}^{m} a_{ijL}^{\alpha} x_i \geq v_L^{\alpha}$ (j=1,2,...……, n)

$$\frac{v_R^{\alpha} + v_L^{\alpha} - \sum_{i=1}^{m}\left(a_{ijL}^{\alpha} + a_{ijR}^{\alpha}\right)x_i}{v_R^{\alpha} - v_L^{\alpha} + \sum_{i=1}^{m}\left(a_{ijR}^{\alpha} - a_{ijL}^{\alpha}\right)x_i} \leq \beta \ \text{(j=1, 2,….. ,n)} \qquad (2)$$

$v_L^{\alpha} \leq v_R^{\alpha}$

$\sum_{i=1}^{m} x_i = 1$

$x_i \geq 0$ (i=1, 2,…,m)

And

Min $\left\{w_R^{\alpha}, \frac{w_L^{\alpha} + w_R^{\alpha}}{2}\right\}$

Subject to $\sum_{j=1}^{n} a_{ijR}^{\alpha} y_j \leq w_R^{\alpha}$ (i=1, 2,…,n)

$$\frac{\sum_{j=1}^{n}\left(a_{ijL}^{\alpha} + a_{ijR}^{\alpha}\right)y_i - (w_L^{\alpha} + w_R^{\alpha})}{w_R^{\alpha} - w_L^{\alpha} + \sum_{j=1}^{n}\left(a_{ijR}^{\alpha} - a_{ijL}^{\alpha}\right)y_i} \leq \beta \ \text{(i=1, 2,…..,n)} \qquad (3)$$

$w_L^{\alpha} \leq w_R^{\alpha}$

$\sum_{j=1}^{n} y_i = 1$

$y_i \geq 0$ (j=1,2,……n)

The equations (2) and (3) are bi-objective linear programming problems (BOLP) on the decision variables $v_L^{\alpha}, v_R^{\alpha}, x_i \ (i = 1,2, \ldots . m)$ and $w_L^{\alpha}, w_R^{\alpha}, y_j \ (j = 1,2, \ldots , n)$.

To solve equations (2) and (3) we get

Max $\left\{\frac{3v_L^{\alpha} + v_R^{\alpha}}{4}\right\}$

Subject to $\sum_{i=1}^{m} a_{ijL}^{\alpha} x_i \geq v_L^{\alpha} \ (j = 1,2, \ldots . n)$

$$\sum_{i=1}^{m}\{(1+\beta)\, a_{ijR}^{\alpha} + (1-\beta) a_{ijL}^{\alpha}\} x_i \geq (1+\beta) v_L^{\alpha} + (1-\beta) v_R^{\alpha} (j = 1,2, \ldots . n)$$

$$v_L^{\alpha} \leq v_R^{\alpha} \qquad (4)$$

$\sum_{i=1}^{m} x_i = 1$

IJMDRR
E- ISSN –2395-1885
ISSN -2395-1877

$x_i \geq 0 (i = 1,2, \ldots . m)$

And

Min $\left\{\frac{3w_R^\alpha + w_L^\alpha}{4}\right\}$

Subject to $\sum_{j=1}^{n} a_{ijR}^{\alpha} y_j \leq w_R^{\alpha} \ (i = 1,2, \ldots . m)$

$$\sum_{j=1}^{n}\{(1+\beta) a_{ijL}^{\alpha} + (1-\beta) a_{ijR}^{\alpha}\} y_j \leq (1-\beta) w_L^{\alpha} + (1+\beta) w_R^{\alpha} (i = 1,2, \ldots . m)$$

$$w_L^{\alpha} \leq w_R^{\alpha} \quad (5)$$

$$\sum_{j=1}^{n} y_j = 1$$

$$y \geq 0 (j = 1,2, \ldots . n)$$

Now, if all TFNs $\tilde{a}_{ij} = (\grave{a}, a, \acute{a})$ are approximated to its α-cut interval $\tilde{a}_{ij}^{\alpha} = [a_{ijL}^{\alpha}, a_{ijR}^{\alpha}]$ we have $a_{ijL}^{\alpha} = \grave{a}_{ij} + \alpha(a_{ij} - \grave{a}_{ij})$ and $a_{ijR}^{\alpha} = \acute{a}_{ij} - \alpha(\acute{a}_{ij} - a_{ij})$

Therefore, the above two liner programming problems reduced to

Max $\left\{\frac{3v_L^\alpha + v_R^\alpha}{4}\right\}$

Subject to $\sum_{i=1}^{m}\{\grave{a}_{ij} + \alpha(a_{ij} - \grave{a}_{ij})\} x_i \geq v_L^{\alpha} \ (j = 1,2, \ldots n)$

$$\sum_{i=1}^{m}[(1+\beta)\{\acute{a}_{ij} - \alpha(\acute{a}_{ij} - a_{ij})\} + (1-\beta)\{\grave{a}_{ij} + \alpha(a_{ij} - \grave{a}_{ij})\}] x_i$$

$$\geq (1+\beta) v_L^{\alpha} + (1-\beta) v_R^{\alpha} \quad (j=1,2,\ldots,n) \quad (6)$$

$$v_L^{\alpha} \leq v_R^{\alpha}$$

$$\sum_{i=1}^{m} x_i = 1$$

$$x_i \geq 0 (i = 1,2, \ldots . m)$$

and

Min $\left\{\frac{3w_R^\alpha + w_L^\alpha}{4}\right\}$

Subject to $\sum_{j=1}^{n}\{\acute{a}_{ij} + \alpha(\acute{a}_{ij} - a_{ij})\} y_i \leq w_R^{\alpha} \ (i = 1,2, \ldots m)$

$$\sum_{j=1}^{n}[(1+\beta)\{\grave{a}_{ij} - \alpha(a_{ij} - \grave{a}_{ij})\} + (1-\beta)\{\acute{a}_{ij} + \alpha(\acute{a}_{ij} - a_{ij})\}] y_j$$

$$\leq (1-\beta) w_L^{\alpha} + (1-\beta) w_R^{\alpha} \quad (i=1,2,\ldots,m) \quad (7)$$

$$w_L^{\alpha} \leq w_R^{\alpha}$$

$$\sum_{j=1}^{n} y_j = 1$$

$$y \geq 0 (j = 1,2, \ldots . n)$$

where the parameters α and β are given by the players/decision makers.

We take β = 0, which indicates that the inequality constraints. For given parameter α, using existing simplex method for linear programming problems, optimal solutions of equations (6) and (7) can be obtained. We denote them as ($x^*, v_L^{\alpha^*}, v_R^{\alpha^*}$) and ($y^*, w_L^{\alpha^*}, w_R^{\alpha^*}$) respectively.

Thus the optimal strategy x^* and the corresponding upper and lower bounds of α-cut of the value of the game, $\ddot{v}^*$ for player I is obtained for given α. Similarly, for player II, optimal strategy y^* and the corresponding upper and lower bounds of α-cut of the value of the game, $\ddot{w}^*$ for player II is obtained for given α.

4. Numerical Example

Suppose that there are two companies I and II to enhance the market share of a new product by competing in advertising. The two companies are considering two different strategies to increase market share: strategy I (adv. by TV), II (adv. by Newspaper). Here it is assumed that the targeted market is fixed, (i.e) the market share of the one company increases while the market share of the other company decreases and also each company puts all its advertisements in one. The above problem may be regarded as matrix game. Namely, the company I and II are considered as players I and II respectively. The marketing research department of company I establish the following pay-off matrix using the trapezoidal numbers we have

Adv.by TV adv.by newspaper

$$\ddot{A} = \begin{matrix} adv.by\ TV \\ adv.by\ newspaper \end{matrix} \begin{pmatrix} (160,175,175,190) & (140,145,145,160) \\ (80,85,85,105) & (160,170,170,190) \end{pmatrix}$$

where the element (160,175,175,190) in the matrix $\ddot{A}$ indicates that the sales amount of the company I increases by "about 175" units when the company I and II use the strategy I (adv. by TV) simultaneously. The other elements in the matrix $\ddot{A}$ can be explained similarly.

According to the equations (6) and (7) we get the linear programming models as

$$\text{Max } \left\{\frac{3v_L^{\alpha} + v_R^{\alpha}}{4}\right\}$$

Subject to $(160+15\alpha)\, x_1 + (180+5\alpha)x_2 \geq v_L^{\alpha}$

$(140+5\alpha)x_1 + (160+10\alpha)x_2 \geq v_L^{\alpha}$

$$\{(1+\beta)(190-15\alpha) + (1-\beta)(160+10\alpha)\}x_1 + \{(1+\beta)(105-20\alpha) + (1-\beta)(80+5\alpha)\}\, x_2 \geq (1+\beta)v_L^{\alpha} + (1-\beta)v_R^{\alpha} \quad (8)$$

$$\{(1+\beta)(190-15\alpha) + (1-\beta)(160+10\alpha)\}x_1 + \{(1+\beta)(100-20\alpha) + (1-\beta)(160+15\alpha)\}\, x_2 \geq (1+\beta)v_L^{\alpha} + (1-\beta)v_R^{\alpha}$$

$v_L^{\alpha} \leq v_R^{\alpha}$

$x_1 + x_2 = 1$

$x_1, x_2 \geq 0$

And

$$\text{Min } \left\{\frac{3w_R^{\alpha} + w_L^{\alpha}}{4}\right\}$$

Subject to $(190-15\alpha)\, y_1 + (160-5\alpha) y_2 \leq w_R^\alpha$

$(105-20\alpha) y_1 + (190-20\alpha) y_2 \leq w_R^\alpha$

$$\{(1+\beta)(160+10\alpha)+(1-\beta)(190-15\alpha)\}y_1+\{(1+\beta)(140+5\alpha)+(1-\beta)(160-20\alpha)\}\,y_2 \geq (1+\beta)w_L^\alpha + (1-\beta)w_R^\alpha \quad (9)$$

$$\{(1+\beta)(80+5\alpha)+(1-\beta)(105-20\alpha)\}y_1+\{(1+\beta)(160+10\alpha)+(1-\beta)(190-20\alpha)\}\,y_2 \geq (1-\beta)w_L^\alpha + (1-\beta)w_R^\alpha$$

$w_L^\alpha \leq w_R^\alpha$

$y_1 + y_2 = 1$

$y_1, y_2 \geq 0$

β = 0 indicates that the inequality constraints. For given α we can solve the above two equations (8) and (9). The upper and lower bounds of α-cut sets of the value of the game $\tilde{v}^*$(respectively, $\tilde{w}^*$) for player I (respectively, player II) and the corresponding optimal mixed strategies for any $\alpha \in [0, 1]$ are shown in the following table.

Table 1: Solution of the game $\widetilde{FG}$ for different values of α.

α	x_1^*	x_2^*	y_1^*	y_2^*	$v_L^{\alpha^*}$	$v_R^{\alpha^*}$	$w_L^{\alpha^*}$	$w_R^{\alpha^*}$
0	0.84716	0.1538	0.2609	0.7391	143.00	181.5	157.82	167.82
0.1	0.84521	0.1548	0.2570	0.7482	143.67	180.1	156.02	166.04
0.2	0.84582	0.1542	0.2428	0.7551	144.02	178.69	156.25	166.84
0.3	0.84526	0.15412	0.2378	0.7621	144.96	178.58	155.11	165.27
0.4	0.84420	0.1550	0.2353	0.7647	145.54	176.22	155.00	159.00
0.5	0.70998	0.2900	0.2521	0.7478	152.02	149.02	144.19	158.00
0.6	0.70241	0.2900	0.2521	0.7475	152.01	148.01	143.00	155.02
0.7	0.75560	0.2444	0.2532	0.7468	149.24	172.39	143.61	154.47
0.8	0.83842	0.1628	0.2638	0.7361	149.01	168.52	145.83	151.57
0.9	0.83813	0.1619	0.2511	0.7489	148.46	168.84	150.65	150.65
1	0.82802	0.1624	0.2511	0.7489	147.93	158.42	158.75	158.75

In the above table the optimal solution for the above two problems (6) and (7) for different values of α. In particular, for α= 0.5, the optimal strategies for player I and player II are
x^* = (0.7099, 0.2900) and y^* = (0.2521, 0.7476) and the cut set of the value of the game for player I and player II are the intervals [152.02, 149.02] and [144.1, 158], respectively.

Table 1 shows that larger the αvalues, lower the degree of uncertainty of the value of the game for both players. Moreover, when α= 0 the cut set of the value of the game of the player I and II are the intervals [143, 181.5] and [157.8, 167.8] respectively, which are the widest.

Thus, in this example the value of the game for player I falls outside of the interval [143, 181.5]. Again for α= 1 the value of the game for player I is 147.9, which is the most likely value. Similarly, the value of the game for player II never falls outside of the interval [157.8, 167.8] and the most likely value is 158.7. Therefore, the approximate values of the game for players I and II are obtained as follows:

$\tilde{v}^*$ = (143, 147.9, 181.5) and

$\tilde{w}^*$ = (157.8, 158.7, 167.8)

respectively, which are TFNs. It means that the sales amount increases of the company I is “approximately 147.9”. In other words, company I’s minimum reward is 143 while his maximum reward is181.5. Similar interpretation can also be given to player II.

4.1. Results obtained by Li’s Approach:

Consider the Li approach [10] the value of the game for player I and player II, respectively as TFNs, denoted by $\tilde{v}^l = (\check{v}^l, v^l, \hat{v}^l)$ and $\tilde{w}^l = (\check{w}^l, w^l, \hat{w}^l)$. Then, player I’s LP problem are constructed as follows

$$\text{Max } \{v^l\}$$

$$\begin{aligned} \text{Subject to } \quad & 160x_1 + 80x_2 \geq \check{v}^l \\ & 140x_1 + 160x_2 \geq \check{v}^l \\ & 175x_1 + 85x_2 \geq v^l \\ & 145x_1 + 170x_2 \geq v^l \\ & 190x_1 + 105x_2 \geq \hat{v}^l \\ & 160x_1 + 190x_2 \geq \hat{v}^l \\ & x_1 + x_2 = 1 \\ & \check{v}^l \leq v^l \leq \hat{v}^l \\ & x_1, x_2 \geq 0 \end{aligned} \qquad (10)$$

Solving (10) by using the simplex method of linear programming we obtain the optimal solution $(x^{*l}, \check{v}^{0l}, v^{*l}, \hat{v}^{0l})$, where x^{*l} = (0.823, 0.210), $\check{v}^{0l}$ =144, v^{*l} =155, $\hat{v}^{0l}$=166

Now, player I’s LP problem are constructed as follows:

$$\text{Max } \{\check{v}^l\}$$

$$\begin{aligned} & \check{v}^l \leq 160 * 0.823 + 80 * 0.210 \\ & \check{v}^l \leq 140 * 0.823 + 160 * 0.210 \\ & \check{v}^l \leq 155 \end{aligned} \qquad (11)$$

$$\text{Max } \{\hat{v}^l\}$$

$$\begin{aligned} & \hat{v}^l \leq 190 * 0.823 + 170 * 0.210 \\ & \hat{v}^l \leq 160 * 0.823 + 190 * 0.210 \\ & \hat{v}^l \geq 155 \end{aligned} \qquad (12)$$

Equations (11) and (12) have the optimal solutions $\check{v}^{*l} = 148.48$ and $\hat{v}^{*l}$ =171.58. Therefore, optimal mixed strategy and corresponding value of the game for player I are x^{*l}= (0.823, 0.210) and $\tilde{v}^{*l}$ = (148.48, 155, 171.58), respectively.

Similarly, the optimal mixed strategy and corresponding value of the game for player II are y^{*l}= (0.2105, 0.823) and $\tilde{w}^{*l}$= (148.24, 155.01, 176.05), respectively.

5. Conclusion

In this paper, two-person zero-sum matrix game is considered where each elements of the pay-off matrix is a TrFN and a new approach is derived to solve such games based on α-cut sets of TrFNs. Our approach is based on Interval programming and weighted average approach which is thereby improving the game value of the matrix. The main advantage of this method is that besides unifying the fuzzy matrix game theory it does not require any defuzzification function or any specification of aspiration levels which otherwise may be difficult to decide in practice.

IJMDRR
E- ISSN –2395-1885
ISSN -2395-1877

References

1. Bector, C.R., S. Chandra, On Duality in Linear Programming with Fuzzy Environment, Fuzzy Sets and Systems, 125(3) (2002), 317-325.
2. Bector, C.R., S. Chandra, V. Vijay, Matrix Games with Fuzzy Goals and Fuzzy Linear Programming duality, Fuzzy Optimization and Decision Making, 3(3) (2004a), 255-269.
3. Bector, C.R., S. Chandra, V. Vijay, Duality in Linear Programming with Fuzzy parameters and Matrix Games with Fuzzy Payoffs, Fuzzy Sets and Systems, 146(2) (2004b), 253-269.
4. Campos, L., Fuzzy Linear Programming Models to Solve Fuzzy Matrix Games, Fuzzy Sets and Systems, 32(3) (1989), 275-289.
5. Cevikel, A.C., M. Ahlatcioglu, Solutions for Fuzzy Matrix Games, Computers and Mathematics with Applications, 60(3) (2010), 399-410.
6. Dubois, D., H. Prade, Fuzzy Sets and Fuzzy Systems, Theory and Applications, Academic Press, Newyork,(1980).
7. Ishibuchi, H., H. Tanaka, Multi Objective Programming in Optimization of the Interval Function, European Journal of Operational Research, 48(1990), 219-225.
8. Kacher, F., M. Larbani, Existence of Equilibrium Solution for a Non-Cooperative Game with Fuzzy Goals and Parameters, Fuzzy Sets and Systems, 159(3) (2008), 164-176.
9. Li, D.F., A Fuzzy Multi Objective Approach to Solve Fuzzy Matrix Games, The Journal of Fuzzy Mathematics, 7(4) (1999), 907-912.
10. Li, D.F., Lexicographic Method for Matrix Games with Pay-Offs of Triangular FuzzyNumbers, International Journal of Uncertainty, Fuzziness and Knowledge BasedSystems, 16(3) (2008), 371-389
11. Li, D.F., Interval Programming Models for Matrix Games with Interval Payoffs ,Optimization Methods and Software, 27(1) (2012a), 1-16.
12. Li, D.F., A Fast Approach to compute Fuzzy values of Matrix Games with Payoffs Triangular Fuzzy Numbers, European Journal of Operational Research, 223(2012b), 421-429.
13. Li, D.F., An Effective Methodology for Solving Matrix Games with Fuzzy Pay-Offs, IEEETransactions on Cybernatics, 43(2) (2013), 610-621.
14. Liu, S.T., C. Kao, Solution of Fuzzy Matrix Games: An Application of the Extension Principle, International Journal of Intelligent Systems, 22(8) (2007), 891-903.
15. Maeda, T., On Characterization of Equilibrium Strategy of Two-Person Zero-Sum Games with Fuzzy Pay-Offs, Fuzzy Sets and Systems, 139(2) (2004), 283-296.
16. Moore, R.E., Method and Application of Interval Analysis, SIAM: Philadelphia, (1979).
17. Nayak, P.K., M. Pal, Solution of Interval Games using Graphical Method, Tamsui Oxford Journal of Mathematical Sciences, 22(1) (2006), 95-115.
18. Nayak, P.K., M. Pal, Linear Programming Technique to Solve Two-Person Matrix Games with Interval pay-offs, Asia-Pacific Journal of Operational Research, 26(2)(2009), 285-305.
19. Neumann, J.V., O. Morgenstern, Theory of Games and Economic Behaviour, Princeton University Press, Princeton: New Jersey, (1947).
20. Owen, G., Game Theory, Academic Press: San Diego, (1995).
21. Sakawa, M., I. Nishizaki, Max-Min Solution for Fuzzy Multi Objective Matrix Games, Fuzzy Sets and Systems, 67(1) (1994), 53-59.
22. Vijay.V., S. Chandra, C. R. Bector, Matrix Games with Fuzzy Goals and Fuzzy Payoffs, Omega, 33(5) (2005), 425-429.
23. Vijay, V., A. Mehra, S. Chandra, C. R. Bector, Fuzzy Matrix Games via a Fuzzy Relation Approach, Fuzzy Optimization and Decision Making, 6(4) (2007), 299-314.
24. Zadeh, L.A., Fuzzy sets, Information and Control, 8(3) (1965), 338-352.