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HOMOTOPY PERTURBATION METHOD AND ADOMIAN DECOMPOSITION METHOD FOR VISCOUS FLOW AND HEAT TRANSFER OVER A NONLINEARLY STRETCHING SHEET

Mrs. P. Asvini*   Dr. M. Palanivelrajan**
**Ph.D. (Part-Time) Research Scholar, Department of Mathematics, Dravidian University,Kuppam,Andhra Pradesh*
***Assistant Professor ,Department of Mathematics, Government Arts College,Paramakudi. Tamil Nadu.*

Abstract
This paper analyzes Adomian decomposition method and the Homotopy perturbation method as two powerful methods which consider the approximate solution of a viscous flow and heat transfer over a nonlinearly stretching sheet as an infinite series usually converging to the accurate solution. The paper also discusses theoretical analysis of the two methods which compared with numerical solution.

Keywords: Unsteady Flow, Boundary Layer Flow, Stretching Surface, Heat Transfer, Homotopy Perturbation Method, Adomian Decomposition Method (ADM) And Padé Approximants.

1. Introduction

The study of two-dimensional boundary layer flow due to a stretching surface is important in a variety of engineering applications such as cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of paper and plastic sheets. In all these cases, a study of flow field and heat transfer can be of significant importance since the quality of the final product depends on skin friction coefficient and surface heat transfer rate.

The problem of heat transfer from a boundary layer flow driven by a continuous moving surface is of importance in a number of industrial manufacturing processes. Several authors have been analysed in various aspects of the pioneering work of Sakiadis [14,15]. Crane [7] has investigated the steady boundary layer flow due to stretching with linear velocity.Vleggaar et al. [17] have analysed the stretching problem with constant surface temperature and Soundalgekar et al. [16] have analysed the constant surface velocity.

Perturbation techniques are based on the existence of small or large parameters, the so-called perturbation quantity. Unfortunately, many nonlinear problems in science and engineering do not contain those kinds of perturbation quantities. Therefore, many different methods have recently introduced some ways to eliminate the small parameter. One of the semi exact methods which do not need small parameters is the Homotopy perturbation method.

The Homotopy perturbation method was developed and improved first by He in 1998.The method yields a very rapid convergence of the solution series in most of cases. The HPM proved its capability to solve a large class of nonlinear problems efficiently, accurately, and easily with approximations convergence very rapidly to solution. Usually, few iterations lead to high-accuracy solution.

Recently, this method is being employed for many researches in engineering sciences. He’s Homotopy perturbation method is applied to obtain approximate analytical solutions for the motion of a spherical particle in a plane couette flow Jalaal et al. [16]. Jalaal et al. [17] showed the effectiveness of HPM for unsteady motion of a spherical particle falling in a Newtonian fluid. Ghotbi et al. [18] used HPM to approximate the solution of the ratio-dependent predatorprey system with constant effort prey harvesting. Homotopy perturbation method is also used for solving nonlinear MHD Jeffery Hamel problem by Moghimi et al. [19]. Recently, Ganji et al. studied the steady-state flow of a Hagen-Poiseuille model in a circular pipe and entropy generation due to fluid friction and heat transfer using HPM [20].

2. Formulation of the Problem

We consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y = 0, the flow being confined to y > 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The basic boundary layer equations that govern momentum and energy respectively are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

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$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\sim\frac{\partial^2 T}{\partial y^2}\frac{\neg}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

subject to the boundary conditions are

$$\begin{aligned}&u_w(x)=Cx^n, v=0\\&u\to 0 \quad y\to\infty\\&T=T_w \text{ at } y=0;\ T\to T_\infty \text{ as } y\to\infty\end{aligned} \quad (4)$$

where *(x,y)* denotes the Cartesian coordinates along the sheet and normal to it, *u* and *v* are the velocity components of the fluid in the x and y directions, respectively, and *v* is the kinematic viscosity. *C* and *n* are parameters related to the surface stretching speed. c_p and $\sim$ are the specific heat of the fluid at constant pressure and the thermal diffusivity respectively.

The equation of continuity is satisfied if we choose a stream function ψ(x, y) such that

$$u=\frac{\partial\psi}{\partial y}, \qquad v=\frac{-\partial\psi}{\partial x}.$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables:

$$\begin{aligned}&¦=y\sqrt{\frac{C(n+1)}{2v}}x^{\frac{n-1}{2}}\\&u=Cx^n f'(¦),\\&v=-\sqrt{\frac{Cv(n+1)}{2}}x^{\frac{n-1}{2}}\left[f+\frac{n-1}{n+1}¦ f'\right]\end{aligned} \quad (5)$$

Substituting (5) into (2) and (3), we obtain the following set of ordinary differential equations:

$$f'''+ff''-(f')^2\left(\frac{2n}{n+1}\right)=0 \quad (6)$$

$$¯''+Prf¯'+PrEc(f'')^2=0 \quad (7)$$

The boundary conditions (4) now become

$$\begin{aligned}&\eta=0 \quad : \quad f=0, \quad f'=1, \quad \theta=1\\&\eta\to\infty \ : \quad f'=0, \quad \theta=0\end{aligned} \quad (8)$$

where the primes denote differentiation with respect to η

$E_c=\frac{u_w^2}{c_p(T_w-T_\infty)}$ is the Eckert number, $Pr\left(=\frac{\neg}{\sim}\right)$ is the Prandtl number Further, the constants T_w, T_∞ denote the temperature at the wall and at a large distance from the wall, respectively.

3. Adomian Decomposition Method

To solve the system of coupled ODEs using Adomian decomposition method, rearranging (6) and (7) as follows

$$f'''=-ff''-f'^2\frac{2n}{n+1} \quad (9)$$

$$\theta''=-Pr\left[f\theta'+Ec(f'')^2\right] \quad (10)$$

While applying the standard procedure of ADM
Eqs (9) and (10) becomes

$$L_1 f = L_1\left(-ff''- f'^2 \frac{2n}{n+1}\right) \tag{11}$$

$$L_2\theta = -Pr\, L_2\left(\left[f\theta'+Ec(f'')^2\right]\right) \tag{12}$$

where

$L_1 = \frac{d^3}{d\eta^3}$ and inverse operator $L_1^{-1}(.) = \int_0^\eta\int_0^\eta\int_0^\eta (.)d\eta d\eta d\eta$ and

$L_2 = \frac{d^2}{d\eta^2}$ and inverse operator $L_2^{-1}(.) = \int_0^\eta\int_0^\eta (.)d\eta d\eta$

Applying the inverse operator on both sides of (11) and (12)

$$L_1^{-1}L_1 f = L_1^{-1}\left(-ff''- f'^2 \frac{2n}{n+1}\right) \tag{13}$$

$$L_2^{-1}L_2\theta = -Pr\, L_2^{-1}\left(\left[f\theta'+Ec(f'')^2\right]\right) \tag{14}$$

Simplifying Eqs (13) and (14) we get

$$f(\eta) = \eta + \frac{\alpha_1\eta^2}{2} + \int_0^\eta\int_0^\eta\int_0^\eta\left[-N_1(f) - N_2(f)\frac{2n}{n+1}\right]d\eta d\eta d\eta \tag{15}$$

and

$$\theta(\eta) = \alpha_2 - \eta - Pr\int_0^\eta\int_0^\eta\left[N_3(f,\theta) + EcN_4(f)\right]d\eta d\eta \tag{16}$$

Where $\alpha_1 = f''(0)$ and $\alpha_2 = \theta(0)$ are to be determined from the boundary conditions at infinity in (8). The nonlinear terms ff'', f'^2 and $f\theta'$ can be decomposed as Adomian polynomials $\sum_{n=0}^{\infty} B_n, \sum_{n=0}^{\infty} C_n, \sum_{n=0}^{\infty} D_n$ and $\sum_{n=0}^{\infty} E_n$ as follows

$$N_1(f) = \sum_{n=0}^{\infty} B_n = ff'' \tag{17}$$

$$N_2(f) = \sum_{n=0}^{\infty} C_n = (f')^2 \tag{18}$$

$$N_3(f,\theta) = \sum_{n=0}^{\infty} D_n = f\theta' \tag{19}$$

$$N_4(f,\theta) = \sum_{n=0}^{\infty} E_n = (f'')^2 \tag{20}$$

Where $B_n(f_0, f_1,..., f_n)$ $C_n(f_0, f_1,..., f_n)$ and $D_n(f_0, f_1,..., f_n,\theta_0,\theta_1,...,\theta_n)$ $E_n(f_0, f_1,..., f_n)$ are the so called Adomian polynomials. In the Adomian decomposition method [1] f and θ can be expanded as the infinite series

$$f(\eta) = \sum_{n=0}^{\infty} f_n = f_0 + f_1 + f_2 + ... + f_m + ...$$

$$\theta(\eta) = \sum_{n=0}^{\infty}\theta_n = \theta_0 + \theta_1 + \theta_2 + ... + \theta_m + ... \tag{21}$$

Substituting (17), (18), (19) and (20) into (15) and (16) gives

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$$\sum_{n=0}^{\infty} f_n(\eta)=\eta+\frac{\alpha_1\eta^2}{2}+\int_0^{\eta}\int_0^{\eta}\int_0^{\eta}\left[-\sum_{n=0}^{\infty}B_n-\frac{2n}{n+1}\sum_{n=0}^{\infty}C_n\right]d\eta d\eta d\eta \tag{22}$$

and

$$\sum_{n=0}^{\infty}\theta_n(\eta)=\alpha_2-\eta-Pr\int_0^{\eta}\int_0^{\eta}\left[\sum_{n=0}^{\infty}D_n+Ec\sum_{n=0}^{\infty}E_n\right]d\eta d\eta \tag{23}$$

Hence, the individual terms of the Adomian series solution of the equation (6)–(8) are provided below by the simple recursive algorithm

$$f_0(\eta)=\eta+\frac{\alpha_1\eta^2}{2} \tag{24}$$

$$\theta_0(\eta)=1+\alpha_2\eta \tag{25}$$

$$f_{n+1}(\eta)=\int_0^{\eta}\int_0^{\eta}\int_0^{\eta}\left[-B_n-C_n\right]d\eta d\eta d\eta \tag{26}$$

$$\theta_{n+1}(\eta)=-Pr\int_0^{\eta}\int_0^{\eta}\left[D_n+EcE_n\right]d\eta d\eta \tag{27}$$

For numerical calculation, we choose the m-term approximation of $f(\eta)$ and $\theta(\eta)$ as $\Phi_m(\eta)=\sum_{n=0}^{m-1} f_n(\eta)$ and

$$\Omega_m(\eta)=\sum_{n=0}^{m-1}\theta_n(\eta)$$

The recursive algorithms (24)–(27) were programmed in the MATLAB. The research delivered upto 15th term of approximations to both $f(\eta)$ and $\theta(\eta)$. Given below are only the first few terms due to lack of space.

$$f_0=\eta+\frac{\alpha_1\eta^2}{2}$$

$$f_1=\left(\frac{1}{3}-\frac{1}{3n+3}\right)\eta^3+\left(\frac{\alpha_1}{8}-\frac{\alpha_1}{6n+6}\right)\eta^4+\left(\frac{\alpha_1^2}{40}-\frac{\alpha_1^2}{30n+30}\right)\eta^5$$

etc.,
and

$$\theta_0=1+\alpha_2\eta$$

$$\theta_1=-Pr\left[\left(\frac{Ec\alpha_1^2}{2}\right)\eta^2+\left(\frac{\alpha_2}{6}\right)\eta^3+\left(\frac{\alpha_1\alpha_2}{24}\right)\eta^4\right]$$

etc.,

4. Homotopy Perturbation Method

4.1 Basic Concepts of HPM

consider the following nonlinear differential equation

$$A(u)-f(r)=0, r\in\Omega \tag{28}$$

Considering the boundary conditions of:

$$B\left(u,\frac{\partial u}{\partial n}\right)=0, r\in\Gamma \tag{29}$$

where A is a general differential operator, B is a boundary operator, *f(r)* is a known analytical function, and Γ is the boundary of the domain Ω.

The operator A can be divided into two parts of L and N, where L is the linear part, while N is a nonlinear one Equation (28) therefore can be rewritten as follows:

$$L(u)+N(u)-f(r)=0, \tag{30}$$

By the Homotopy technique, we construct a Homotopy as $v(r,p):\Omega\times[0,1]\rightarrow\Re$ which satisfies:

$$H(v,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)]=0,\ p\in[0,1]\, r\in\Omega, \tag{31}$$

Where $p\in[0,1]$ is an embedding parameter and u_0 is an initial approximation of Eqs (30)

This satisfies the boundary conditions.

Consider Eqs (31),

$$\begin{cases} H(v,0)=L(v)-L(u_0)=0 \\ H(v,1)=A(v)-f(r)=0. \end{cases} \tag{32}$$

The changing process of *p* from zero to unity is just that of *v(r, p)* from $u_0(r)$ *to u(r)*.

In topology, this is called deformation, $L(v)-L(u_0)$ and $A(v)-f(r)$ are called Homotopy. According to HPM, we can first use the embedding parameter *p* as “small parameter”, and assume that the solution of Eqs (31) can be written as a power series in *p*:

$$v=v_0+pv_1+p^2v_2+\ldots, \tag{33}$$

Setting *p*=1 results in the approximate solution of Eqs (31)

$$u=\lim_{p\rightarrow 1}v=v_0+v_1+v_2+\ldots, \tag{34}$$

The combination of the perturbation method and the Homotopy method is called the HPM, which lacks the limitations of the traditional perturbation methods although this technique has full advantages of the traditional perturbation techniques. The series Eqs (34) is convergent for most cases. However, the convergence rate depends on the nonlinear operator *A(v)*.

4.2 Homotopy Perturbation Solutions

According to HPM, we can construct a Homotopy of Equations (6) and (7) as

$$H(F,p)=(1-p)\left(\frac{d^3}{d\eta^3}F(\eta)\right)+p\left[F(\eta)\frac{d^2}{d\eta^2}F(\eta)-\frac{2n}{n+1}\left(\frac{d}{d\eta}F(\eta)\right)^2\right] \tag{35}$$

and

$$H(\theta,p)=(1-p)\left(\frac{d^2}{d\eta^2}\theta(\eta)\right)+\Pr p\left[F(\eta)\frac{d}{d\eta}\theta(\eta)+Ec\left(\frac{d^2}{d\eta^2}F(\eta)\right)^2\right] \tag{36}$$

We consider

$$F(\eta)=F_0(\eta)+F_1(\eta)p+F_2(\eta)p^2+F_3(\eta)p^3+\ldots\ldots, \tag{37}$$

$$\theta(\eta)=\theta_0(\eta)+\theta_1(\eta)p+\theta_2(\eta)p^2+\theta_3(\eta)p^3+\ldots\ldots, \tag{38}$$

substituting *F* and *θ* from Equations (37) and (38) into Equations (35) and (36) and some simplification and rearranging based on powers of *p*-terms

$$p^0:\begin{cases} \dfrac{d^3}{d\eta^3}F_0(\eta)=0 \\ \dfrac{d^2}{d\eta^2}\theta_0(\eta)=0 \end{cases} \tag{39}$$

$$F_0(0)=0, F_0'(0)=1, F_0''(0)=\gamma_1, \theta_0(0)=1, \theta_0'(0)=\gamma_2, \tag{40}$$

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$$p^1:\begin{cases}\dfrac{1}{n+1}\left[n\dfrac{d^3}{d\eta^3}F_1(\eta)+nF_0(\eta)\dfrac{d^2}{d\eta^2}F_0(\eta)+\dfrac{d^3}{d\eta^3}F_1(\eta)+F_0(\eta)\dfrac{d^2}{d\eta^2}F_0(\eta)-2n\left(\dfrac{d}{d\eta}F_0(\eta)\right)^2\right]=0\\ \dfrac{d^2}{d\eta^2}\theta_1(\eta)+\Pr\left[F_0(\eta)\dfrac{d}{d\eta}\theta_1(\eta)+Ec\left(\dfrac{d^2}{d\eta^2}F_0(\eta)\right)^2\right]=0\end{cases}\tag{41}$$

$$F_0(0)=0, F_0^{'}(0)=0, F_0^{"}(0)=0, \theta_0(0)=0, \theta_0^{'}(0)=0,\tag{42}$$

$$p^2:\begin{cases}\dfrac{1}{n+1}\left[n\dfrac{d^3}{d\eta^3}F_2(\eta)+nF_0(\eta)\dfrac{d^2}{d\eta^2}F_1(\eta)+nF_1(\eta)\dfrac{d^2}{d\eta^2}F_0(\eta)+\dfrac{d^3}{d\eta^3}F_2(\eta)+F_1(\eta)\dfrac{d^2}{d\eta^2}F_0(\eta)-4n\dfrac{d}{d\eta}F_1(\eta)\dfrac{d}{d\eta}F_0(\eta)\right]=0\\ \dfrac{d^2}{d\eta^2}\theta_2(\eta)+\Pr\left[F_1(\eta)\dfrac{d}{d\eta}\theta_0(\eta)+F_0(\eta)\dfrac{d}{d\eta}\theta_1(\eta)+2Ec\dfrac{d^2}{d\eta^2}F_0(\eta)\dfrac{d^2}{d\eta^2}F_1(\eta)\right]=0\end{cases}\tag{43}$$

$$F_0(0)=0, F_0^{'}(0)=0, F_0^{"}(0)=0, \theta_0(0)=0, \theta_0^{'}(0)=0,\tag{44}$$

Solving Eqs (39),(41),(43) with the boundary conditions, Eqs (40),(42),(44) using MATLAB the research delivered upto 15th term of approximations to both $f(\eta)$ and $\theta(\eta)$. Given below are only the first few terms due to lack of space.

$$f_0=\eta+\frac{\alpha_1\eta^2}{2}$$

$$f_1=\left(\frac{1}{3}-\frac{1}{3n+3}\right)\eta^3+\left(\frac{\alpha_1}{8}-\frac{\alpha_1}{6n+6}\right)\eta^4+\left(\frac{\alpha_1^2}{40}-\frac{\alpha_1^2}{30n+30}\right)\eta^5$$

etc.,
and

$$\theta_0=1+\alpha_2\eta$$

$$\theta_1=-Pr\left[\left(\frac{Ec\alpha_1^2}{2}\right)\eta^2+\left(\frac{\alpha_2}{6}\right)\eta^3+\left(\frac{\alpha_1\alpha_2}{24}\right)\eta^4\right]$$

etc.,
which is exactly the same as Adomian decomposition solution.

The undetermined values of α_1 and α_2 are calculated from the boundary conditions at infinity in (8). The difficulty at infinity is overcome by employing the diagonal Padé approximants [10] that approximate $f'(\eta)$ and $\theta(\eta)$ using $\Phi_{15}^{'}(\eta)$ and $\Omega_{15}(\eta)$ respectively. The numerical results of α_1 and α_2 from $lim_{\eta\to\infty}\Phi_{15}^{'}(\eta)=0$ and $lim_{\eta\to\infty}\Omega_{15}(\eta)=0$ for selected *m* in the range from 4 to 8 are shown in the Tables below.

5. Results and Discussion

Table 1

The velocity gradient ($\alpha_1 = f''(0)$) for various values of n using HPM-Padé and ADM-Padé techniques

A	Present Result				R Cortell [7]
	ADM-Padé		HPM-Padé		
	$-f''(0)$		$-f''(0)$		$-f''(0)$
	[7/7]	[8/8]	[7/7]	[8/8]	
0	0.62821	0.62802	0.62725	0.62754	0.62754
0.2	0.76775	0.76597	0.76675	0.76664	0.76675

0.5	0.88999	0.88974	0.88889	0.88947	0.88947
0.75	0.95861	0.95378	0.95382	0.95012	0.95378
1	1.05	1	1.066	1.023	1
1.5	1.06789	1.06158	1.06201	1.06158	1.06158
3	1.14986	1.14991	1.14858	1.14789	1.14858
7	1.21874	1.21684	1.21651	1.21598	1.21684
10	1.23357	1.23487	1.23488	1.23584	1.23487
20	1.25836	1.25741	1.25689	1.25741	1.25741
100	1.27675	1.27676	1.27675	1.27566	1.27676

Table 2,The velocity gradient ($\sim_2 = {}^{-}{}'(0)$) for various values of *n* with *Ec=0, Pr=1* using HPM-Padé and ADM-Padé techniques

n	$-{}^{-}{}'(0)$	$-{}^{-}{}'(0)$		$-{}^{-}{}'(0)$		
	ADM-Padé of	ADM-Padé of	ADM-Padé of	HPM-Padé of	HPM-Padé of	HPM-Padé of
	[6/6]	[7/7]	[8/8]	[6/6]	[7/7]	[8/8]
0.5	0.52665	0.56199	0.56269	0.52674	0.56187	0.56236
1	0.57104	0.54021	0.54693	0.57123	0.54023	0.54723
3	0.52568	0.52549	0.52793	0.52569	0.52446	0.52793
10	0.47792	0.47576	0.47895	0.47785	0.47653	0.47881

Table 3,The velocity gradient ($\sim_2 = {}^{-}{}'(0)$) for various values of *Ec* at *n=1* and *Pr=1* using HPM-Padé and ADM-Padé techniques

Ec	$-{}^{-}{}'(0)$	$-{}^{-}{}'(0)$		$-{}^{-}{}'(0)$		
	ADM-Padé of	ADM-Padé of	ADM-Padé of	HPM-Padé of	HPM-Padé of	HPM-Padé of
	[6/6]	[7/7]	[8/8]	[6/6]	[7/7]	[8/8]
0	0.57104	0.54021	0.54693	0.57104	0.54021	0.54693
0.2	0.19981	0.19979	0.19927	0.19982	0.19979	0.19929
0.5	0.49988	0.49957	0.4998	0.49987	0.49956	0.4998
1	1	0.99998	0.99995	1	0.99998	0.99995

Table 4,The velocity gradient ($\sim_2 = {}^{-}{}'(0)$) for various values of *Pr* at *n=3* and *Ec=1* using HPM-Padé and ADM-Padé techniques

Ec	$-{}^{-}{}'(0)$			$-{}^{-}{}'(0)$		
	ADM-Padé of	ADM-Padé of	ADM-Padé of	HPM-Padé of	HPM-Padé of	HPM-Padé of
	[6/6]	[7/7]	[8/8]	[6/6]	[7/7]	[8/8]
0.5	0.26181	0.28264	0.28894	0.26181	0.28264	0.28894
0.72	0.42176	0.46637	0.46964	0.42176	0.46687	0.46964
1	0.76478	0.75348	0.75557	0.76478	0.75348	0.75567
2	1.5194	1.4984	1.4991	1.5194	1.4984	1.4995

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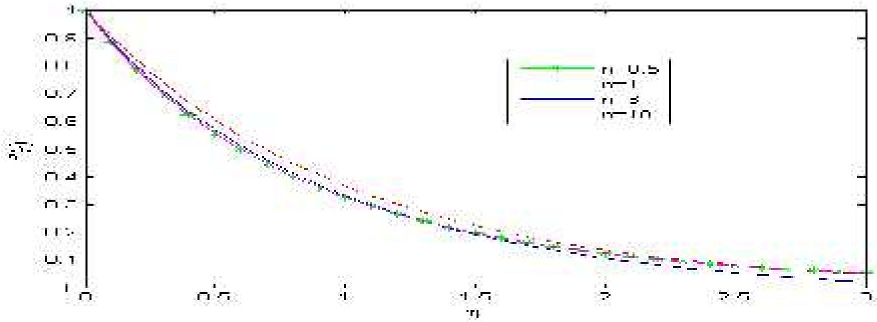

Fig. 1 Velocity profiles $f'(\eta)$ for various values of *n* when *Pr*= 1 and *Ec*=1 Using $\Phi'_{15[7/7]}$.

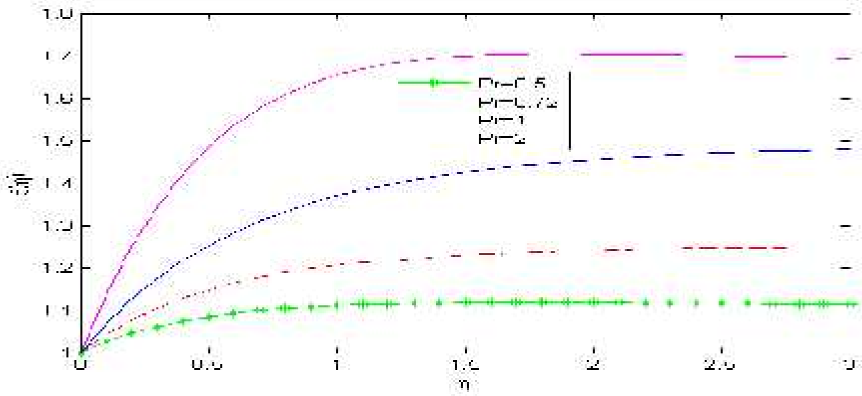

Fig. 2 Temperature profiles $\theta(\eta)$ for various values of Pr at n = 3 and Ec= 1 Using $\Omega_{15[7/7]}$.

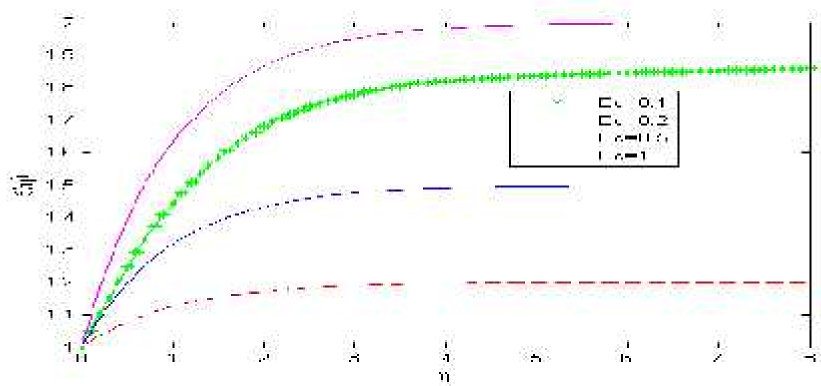

Fig. 3 Temperature profiles $\theta(\eta)$ for various values of Ec at n = 1 and Pr= 1 Using $\Omega_{15[7/7]}$.

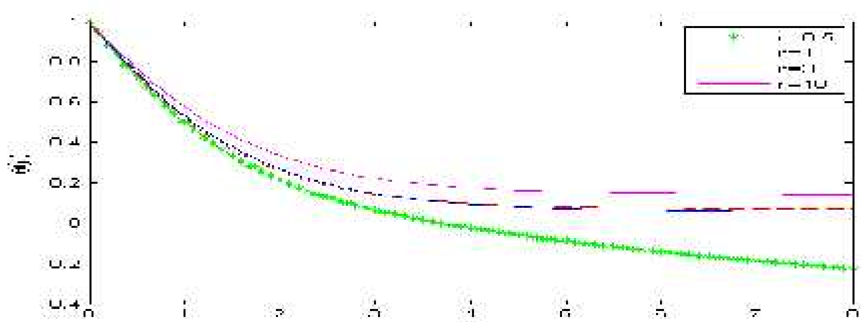

Fig. 4 Temperature profiles $\theta(\eta)$ for various values of n at Ec = 0 and Pr= 1 Using $\Omega_{15[8/8]}$.

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From Fig.1 we note that when unsteadiness parameter *n* increases, the velocity profile decreases. In Figs. 2 and 3 we note that when Prandtl Number (Pr) increases that implies the temperature decreases within the boundary layer for all values of the Prandtl number. This is consistent with the well-known fact that the thermal boundary layer thickness decreases with increasing Prandtl number. In Fig 4 we note that when unsteadiness parameter *n* increases the temperature profiles decreases.

5. Conclusion

The Homotopy perturbation method and Adomian decomposition method is applied to solve a system of two nonlinear ordinary differential equations with a specified boundary condition that describes viscous flow and heat transfer over a nonlinearly stretching sheet. The obtained solutions have matched with the existing numerical result. The Homotopy perturbation method and Adomian decomposition method techniques are very efficient alternative tools to solve nonlinear models with infinite boundary conditions.

References

1. Adomian, G. (1994) “Solving Frontier Problems of Physics”: The Decomposition Method. Kluwer Academic Publication, Boston.
2. Ali, M.E. (1995) “On thermal boundary layer on a power law stretched surface with suction or injection,” Int. J. Heat Mass Flow, Vol. 16, pp. 280–290.
3. Awang Kechil S., Hashim I., (2009) “Approximate analytical solution for MHD stagnation-point flow in porous media”, Communications in Nonlinear Science and Numerical Simulation, Vol. 14, No. 2, pp. 1346–1354.
4. Awang Kechil W.S., Hashim I. (2007), “Non-perturbative solution of free-convective boundary layer equation by Adomian decomposition method,” Physics Letters A, Vol. 363, No. 1–2, pp.110–114.
5. Banks, W.H.H. (1983) “Similarity solutions of the boundary layer equation for a stretching wall,” J. Mec. Theor.Appl. Vol.2, pp.375–392.
6. Boyd, J.P., (1997). “Padé approximant algorithm for solving nonlinear ordinary differential equation boundary value problems on an unbounded domain,”Comput. Phys. Vol.11, pp. 299–303.
7. Crane, L. (1970). “Flow past a stretching plate”. Z. Angew. Mater. Phys. Vol.21, pp. 645–647.
8. Elbashbeshy, (1998) “Heat transfer over a stretching surface with variable heat flux,” J. Phys. D: Appl. Phys.Vol.31, pp.1951–1955.
9. Elsayed, M.A. Elbashbeshy, Dalia A. Aldawody (2010)**,**“Heat transfer over an unsteady stretching surface with variable heat flux in the presence of a heat source or sink”,Computers & Mathematics with Applications,Vol.60,No.10, pp.2806–2811.
10. Grubka, L.J. and K.M. Bobba, (1985),“Heat transfer characteristics of a continuous stretching surface with variable temperature,” J. Heat Transfer Vol.107, pp. 248–250.
11. Hayat T., Hussain Q., Javed T. (2009),“The modified decomposition method and Padé approximants for the MHD flow over a non-linear stretching sheet, Nonlinear Analysis Real World Applications,” Vol. 10, No. 2, pp. 966–973.
12. Hashim I., Noorani M.S.M., Batiha B. (2006) “A note on the Adomian decomposition method for the generalized Huxley equation,”Applied Mathematics and Computation, Vol. 181, No. 2, pp. 1439–1445.
13. Ishak, A R. Nazar, I. Pop, (2008) “Heat transfer over an unsteady stretching surface with prescribed heat flux,” Can. J. Phys. Vol.86, pp. 853–855.
14. Sakiadis, B.C. (1961) “Boundary layer behaviour on continuous solid surfaces: I. Boundary layer equations for two dimensional and axisymmetric flow, AIChE J. Vol.7.No.1, pp.26–28.
15. Sakiadis, B.C. (1961), “Boundary layer behaviour on continuous solid surfaces: II. Boundary layer equations on a continuous flat surface,” AIChE J. Vol.7, No.1, pp.221–225.
16. Soundalgekar, V.M. T.V. Ramana, (1980), “Heat transfer past a continuous moving plate with variable temperature,”Warme-Und Stoffubertragung.Vol**.**14, pp. 91–93.
17. Vleggaar, J. (1977), “laminar boundary layer behaviour on continuous accelerating surfaces,” Chem. Eng. Sci.Vol. 32, pp.1517–1525.
18. Wazwaz, A.M. (1997), “A study on a boundary – layer equation arising in an incompressible fluid,” Appl. Math. Comput, Vol.87, pp.199-204.