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SOME PROPERTIES OF INTUITIONTISTIC FUZZY SUBHEMIRINGS OF A HEMIRING

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INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R ; + ; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (R ; + ; .) share the same properties as a ring except that (R ; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b + c) = a. b + a. c and (b + c) .a = b. a + c. a for all a, b and c in R. A semiring R is said to be additively commutative if a + b = b + a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0 + a = a = a + 0 and a. 0 = 0 = 0. a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A.Zadeh[10], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subsets (IFS) was introduced by K.T.Atanassov[4], as a generalization of the notion of fuzzy set. The notion of Fuzzy left *h*-ideals in hemirings with respect to a *s*-norm was introduced in [2].In this paper, we introduce the some Theorems in intuitionistic fuzzy subhemiring of a hemiring.

1.PRELIMINARIES

1.1Definition: Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \rightarrow [0, 1]$.

1.2 Definition: An intuitionistic fuzzy subset (IFS) A in X is defined as an object of the form A = { < x, $\mu_A(x)$, $\nu_A(x)$ > / $x \in X$ }, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.3 Definition: Let R be a hemiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy subhemiring (IFSHR) of R if it satisfies the following conditions:

(i) $\mu_A(x + y) \geq \min\{ \mu_A(x), \mu_A(y) \}$,
(ii) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y) \}$,
(iii) $\nu_A(x + y) \leq \max\{ \nu_A(x), \nu_A(y) \}$,
(iv) $\nu_A(xy) \leq \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in R.

1.4 Definition: Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as AxB ={⟨ (x, y), $\mu_{AxB}(x,y)$, $\nu_{AxB}(x,y)$ ⟩ / for all x in G and y in H }, where $\mu_{AxB}(x, y) = \min \{ \mu_A(x), \mu_B(y)\}$ and $\nu_{AxB}(x, y) = \max\{ \nu_A(x), \nu_B(y) \}$.

1.5 Definition: Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is an intuitionistic fuzzy relation on A is V given by $\mu_V(x, y) = \min \{ \mu_A(x), \mu_A(y) \}$ and $\nu_V(x, y) = \max \{ \nu_A(x), \nu_A(y) \}$, for all x and y in S.

2. SOME PROPERTIES OF INTUITIONISTIC FUZZY SUBHEMIRING OF A HEMIRING R.

2.1 Theorem: Let (R, +, .) be a hemiring. Intersection of any two intuitionistic fuzzy subhemiring of a hemiring R is an intuitionistic fuzzy subhemiring of R.

Proof: Let A and B be any two intuitionistic fuzzy subhemiring of a hemiring R and let x and y in R. Let A = { (x, $\mu_A(x)$, $\nu_A(x)$) / $x \in R$ } and B = { (x, $\mu_B(x)$, $\nu_B(x)$) / $x \in R$ } and also let C= $A \cap B$ ={(x, $\mu_C(x)$, $\nu_C(x)$) / $x \in R$ }, where $\min \{ \mu_A(x), \mu_B(x) \} = \mu_C(x)$ and $\max\{\nu_A(x), \nu_B(x)\}=\nu_C(x)$. Now, $\mu_C(x+y) = \min\{\mu_A(x+y), \mu_B(x+y)\} \geq \min\{\min\{\mu_A(x), \mu_A(y) \}, \min\{\mu_B(x), \mu_B(y)\}\} = \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\}= \min \{\mu_C(x), \mu_C(y) \}$.Therefore, $\mu_C(x+y) \geq \min\{\mu_C(x), \mu_C(y)\}$, for all x and y in R. And, $\mu_C(xy) = \min\{\mu_A(x y), \mu_B(x y) \}\geq \min\{\min\{\mu_A(x), \mu_A(y) \}, \min\{\mu_B(x), \mu_B(y)\}\} = \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\}= \min\{\mu_C(x), \mu_C(y)\}$. Therefore, $\mu_C(xy) \geq \min\{\mu_C(x), \mu_C(y) \}$, for all x and y in R. Also, $\nu_C(x+y) = \max\{\nu_A(x+y), \nu_B(x+y)\}\leq \max\{\max\{ \nu_A(x), \nu_A(y) \}, \max\{ \nu_B(x), \nu_B(y)\}\}= \max\{ \max\{\nu_A(x), \nu_B(x) \},\max \{ \nu_A(y),\nu_B(y)\} \}= \max\{ \nu_C(x), \nu_C(y)\}$.Therefore, $\nu_C(x+y) \leq \max\{\nu_C(x), \nu_C(y)\}$, for all x and y in R. And, $\nu_C(x y) = \max\{\nu_A(xy), \nu_B(xy)\}\leq \max\{ \max\{\nu_A(x), \nu_A(y)\}, \max\{\nu_B(x), \nu_B(y) \}\}=\max\{\max\{\nu_A(x), \nu_B(x)\}, \max\{\nu_A(y), \nu_B(y)\}\}= \max\{\nu_C(x), \nu_C(y)\}$.Therefore, $\nu_C(xy) \leq \max\{\nu_C(x), \nu_C(y)\}$, for all x and y in R. Therefore, C is an intuitionistic fuzzy subhemiring of a hemiring R. Hence, intersection of any two intuitionistic fuzzy subhemiring of a hemiring R is an intuitionistic fuzzy subhemiring of R.

2.2 Theorem: Let (R, +, .) be a hemiring. The intersection of a family of intuitionistic fuzzy subhemirings of R is an intuitionistic fuzzy subhemiring of R.

Proof: Let $\{V_i : i \in I\}$ be a family of intuitionistic fuzzy subhemirings of a hemiring R and let $A = \bigcap_{i\in I} V_i$. Let x and y in R. Then, $\mu_A(x+y) = \inf_{i\in I} \mu_{Vi}(x+y) \geq \inf_{i\in I} \min\{\mu_{Vi}(x), \mu_{Vi}(y)\} = \min\{\inf_{i\in I} \mu_{Vi}(x), \inf_{i\in I} \mu_{Vi}(y)\} = \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. And, $\mu_A(xy) = \inf_{i\in I} \mu_{Vi}(xy) \geq \inf_{i\in I} \min\{\mu_{Vi}(x), \mu_{Vi}(y)\} = \min\{\inf_{i\in I} \mu_{Vi}(x), \inf_{i\in I} \mu_{Vi}(y)\} = \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R. Also, $\nu_A(x+y) = \sup_{i\in I} \nu_{Vi}(x+y) \leq \sup_{i\in I} \max\{\nu_{Vi}(x), \nu_{Vi}(y)\} = \max\{\sup_{i\in I} \nu_{Vi}(x), \sup_{i\in I} \nu_{Vi}(y)\} = \max\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. And, $\nu_A(xy) = \sup_{i\in I} \nu_{Vi}(xy) \leq \sup_{i\in I} \max\{\nu_{Vi}(x), \nu_{Vi}(y)\} = \max\{\sup_{i\in I} \nu_{Vi}(x), \sup_{i\in I} \nu_{Vi}(y)\} = \max\{\nu_A(x), \nu_A(y)\}$. Therefore, $\nu_A(x\,y) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R. That is, A is an intuitionistic fuzzy subhemiring of a hemiring R. Hence, the intersection of a family of intuitionistic fuzzy subhemirings of R is an intuitionistic fuzzy subhemiring of R.

2.3 Theorem: If A and B are any two intuitionistic fuzzy subhemiring of the hemirings R_1 and R_2 respectively, then AxB is an intuitionistic fuzzy subhemiring of R_1xR_2.

Proof: Let A and B be two intuitionistic fuzzy subhemiring of the hemirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in R_1xR_2. Now, $\mu_{AxB}[(x_1, y_1) + (x_2, y_2)] = \mu_{AxB}(x_1 + x_2, y_1 + y_2) = \min\{\mu_A(x_1 + x_2), \mu_B(y_1 + y_2)\} \geq \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_B(y_1), \mu_B(y_2)\}\} = \min\{\min\{\mu_A(x_1), \mu_B(y_1)\}, \min\{\mu_A(x_2), \mu_B(y_2)\}\} = \min\{\mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\}$. Therefore, $\mu_{AxB}[(x_1, y_1) + (x_2, y_2)] \geq \min\{\mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\}$. Also, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] = \mu_{AxB}(x_1x_2, y_1y_2) = \min\{\mu_A(x_1x_2), \mu_B(y_1y_2)\} \geq \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_B(y_1), \mu_B(y_2)\}\} = \min\{\min\{\mu_A(x_1), \mu_B(y_1)\}, \min\{\mu_A(x_2), \mu_B(y_2)\}\} = \min\{\mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\}$. Therefore, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] \geq \min\{\mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\}$. And, $\nu_{AxB}[(x_1, y_1) + (x_2, y_2)] = \nu_{AxB}(x_1 + x_2, y_1 + y_2) = \max\{\nu_A(x_1 + x_2), \nu_B(y_1 + y_2)\} \leq \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_B(y_1), \nu_B(y_2)\}\} = \max\{\max\{\nu_A(x_1), \nu_B(y_1)\}, \max\{\nu_A(x_2), \nu_B(y_2)\}\} = \max\{\nu_{AxB}(x_1, y_1), \nu_{AxB}(x_2, y_2)\}$.

Therefore, $\nu_{AxB}[(x_1, y_1) + (x_2, y_2)] \leq \max\{\nu_{AxB}(x_1, y_1), \nu_{AxB}(x_2, y_2)\}$. Also, $\nu_{AxB}[(x_1, y_1)(x_2, y_2)] = \nu_{AxB}(x_1x_2, y_1y_2) = \max\{\nu_A(x_1x_2), \nu_B(y_1y_2)\} \leq \max\{\max\{\nu_A(x_1), \nu_A(x_2)\}, \max\{\nu_B(y_1), \nu_B(y_2)\}\} = \max\{\max\{\nu_A(x_1), \nu_B(y_1)\}, \max\{\nu_A(x_2), \nu_B(y_2)\}\} = \max\{\nu_{AxB}(x_1, y_1), \nu_{AxB}(x_2, y_2)\}$. Therefore, $\nu_{AxB}[(x_1, y_1)(x_2, y_2)] \leq \max\{\nu_{AxB}(x_1, y_1), \nu_{AxB}(x_2, y_2)\}$. Hence AxB is an intuitionistic fuzzy subhemiring of hemiring of R_1xR_2.

2.4 Theorem

Let A and B be intuitionistic fuzzy subhemirings of the hemirings R_1 and R_2 respectively. Suppose that e and $e^|$ are the identity element of R_1 and R_2 respectively. If AxB is an intuitionistic fuzzy subhemiring of R_1xR_2, then at least one of the following two statements must hold.

(i) $\mu_B(e^|) \geq \mu_A(x)$ and $\nu_B(e^|) \leq \nu_A(x)$, for all x in R_1,
(ii) $\mu_A(e) \geq \mu_B(y)$ and $\nu_A(e) \leq \nu_B(y)$, for all y in R_2.

Proof

Let AxB be an intuitionistic fuzzy subhemiring of R_1xR_2. By contraposition, suppose that none of the statements (i) and (ii) holds. Then we can find a in R_1 and b in R_2 such that $\mu_A(a) > \mu_B(e^|)$, $\nu_A(a) < \nu_B(e^|)$ and $\mu_B(b) > \mu_A(e)$, $\nu_B(b) < \nu_A(e)$. We have, $\mu_{AxB}(a, b) = \min\{\mu_A(a), \mu_B(b)\} > \min\{\mu_B(e^|), \mu_A(e)\} = \min\{\mu_A(e), \mu_B(e^|)\} = \mu_{AxB}(e, e^|)$. And, $\nu_{AxB}(a, b) = \max\{\nu_A(a), \nu_B(b)\} < \max\{\nu_B(e^|), \nu_A(e)\} = \max\{\nu_A(e), \nu_B(e^|)\} = \nu_{AxB}(e, e^|)$. Thus AxB is not an intuitionistic fuzzy subhemiring of R_1xR_2. Hence either $\mu_B(e^|) \geq \mu_A(x)$ and $\nu_B(e^|) \leq \nu_A(x)$, for all x in R_1 or $\mu_A(e) \geq \mu_B(y)$ and $\nu_A(e) \leq \nu_B(y)$, for all y in R_2.

2.5 Theorem Let A and B be two intuitionistic fuzzy subsets of the hemirings R_1 and R_2 respectively and AxB is an intuitionistic fuzzy subhemiring of R_1xR_2. Then the following are true:

(i) if $\mu_A(x) \leq \mu_B(e^{|})$ and $\nu_A(x) \geq \nu_B(e^{|})$, then A is an intuitionistic fuzzy subhemiring of R_1.

(ii) if $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, then B is an intuitionistic fuzzy subhemiring of R_2.

(iii) either A is an intuitionistic fuzzy subhemiring of R_1 or B is an intuitionistic fuzzy subhemiring of R_2.

Proof: Let AxB be an intuitionistic fuzzy subhemiring of R_1xR_2 and x and y in $R_1$and $e^{|}$ in R_2. Then $(x, e^{|})$ and $(y, e^{|})$ are in R_1xR_2. Now, using the property that $\mu_A(x) \leq \mu_B(e^{|})$ and $\nu_A(x) \geq \nu_B(e^{|})$, for all x in R_1. We get, $\mu_A(x+y) = \min\{ \mu_A(x+y), \mu_B(e^{|}+e^{|})\} = \mu_{AxB}((x+ y), (e^{|}+ e^{|})) = \mu_{AxB}[(x, e^{|}) + (y, e^{|})] \geq \min\{\mu_{AxB}(x, e^{|}), \mu_{AxB}(y, e^{|}) \} = \min\{\min\{\mu_A(x), \mu_B(e^{|}) \}, \min\{\mu_A(y), \mu_B(e^{|}) \}\} = \min\{ \mu_A(x), \mu_A(y)\} \geq \min\{\mu_A(x), \mu_A(y)\}$. Therefore, $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R_1. Also, $\mu_A(xy) = \min\{\mu_A(xy), \mu_B(e^{|}e^{|}) \}= \mu_{AxB}((xy), (e^{|}e^{|}))= \mu_{AxB}[(x, e^{|})(y, e^{|})] \geq \min\{\mu_{AxB}(x, e^{|}), \mu_{AxB}(y, e^{|}) \}= \min\{\min\{ \mu_A(x), \mu_B(e^{|}) \}, \min\{\mu_A(y), \mu_B(e^{|}) \}\} = \min\{\mu_A(x), \mu_A(y) \}$.Therefore, $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y) \}$, for all x and y in R_1.And, $\nu_A(x+y) = \max\{\nu_A(x+y), \nu_B(e^{|}+e^{|}) \}= \nu_{AxB}((x+y), (e^{|}+e^{|})) = \nu_{AxB}[(x, e^{|}) + (y, e^{|})] \leq \max\{\nu_{AxB}(x, e^{|}), \nu_{AxB}(y, e^{|}) \}= \max\{\max\{\nu_A(x), \nu_B(e^{|}) \}, \max\{ \nu_A(y), \nu_B(e^{|}) \}\} = \max\{\nu_A(x), \nu_A(y) \}$. Therefore, $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y) \}$, for all x and y in R_1.

Also, $\nu_A(xy) = \max\{\nu_A(xy), \nu_B(e^{|}e^{|}) \}= \nu_{AxB}((xy), (e^{|}e^{|})) = \nu_{AxB}[(x, e^{|})(y, e^{|})] \leq \max\{\nu_{AxB}(x, e^{|}), \nu_{AxB}(y, e^{|}) \}= \max\{\max\{\nu_A(x) , \nu_B(e^{|}) \}, \max\{\nu_A(y), \nu_B(e^{|}) \}\} = \max\{\nu_A(x), \nu_A(y) \}$.Therefore, $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$, for all x and y in R_1.

Hence A is an intuitionistic fuzzy subhemiring of R_1. Thus (i) is proved.

Now, using the property that $\mu_B(x) \leq \mu_A(e)$ and $\nu_B(x) \geq \nu_A(e)$, for all x in R_2, let x and y in R_2 and e in R_1. Then (e, x) and (e, y) are in R_1xR_2.We get, $\mu_B(x+y) = \min\{\mu_B(x+y), \mu_A(e+e)\}= \min\{\mu_A(e+e), \mu_B(x+y) \}= \mu_{AxB}((e+e), (x+y)) = \mu_{AxB}[(e, x) + (e, y)] \geq \min\{\mu_{AxB}(e, x), \mu_{AxB}(e, y) \}= \min\{\min\{\mu_A(e), \mu_B(x) \}, \min\{\mu_A(e), \mu_B(y) \} = \min\{\mu_B(x), \mu_B(y) \}\geq \min\{\mu_B(x), \mu_B(y) \}$.Therefore, $\mu_B(x+ y) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R_2. Also, $\mu_B(xy) = \min\{\mu_B(xy), \mu_A(ee) \}= \min\{\mu_A(ee), \mu_B(xy) \} = \mu_{AxB}((ee), (xy)) = \mu_{AxB}[(e, x)(e, y)]\geq \min\{\mu_{AxB}(e, x), \mu_{AxB}(e, y)\} =\min\{\min\{\mu_A(e), \mu_B(x) \}, \min\{\mu_A(e), \mu_B(y) \} \}= \min\{\mu_B(x), \mu_B(y)\}$. Therefore, $\mu_B(xy) \geq \min\{\mu_B(x), \mu_B(y)\}$, for all x and y in R_2. And, $\nu_B(x+y) = \max\{\nu_B(x+y), \nu_A(e+e)\}= \max\{\nu_A(e+e), \nu_B(x+y)\}= \nu_{AxB}((e+e), (x+y)) = \nu_{AxB}[(e, x) + (e, y)] \leq \max\{\nu_{AxB}(e, x), \nu_{AxB}(e, y) \}= \max\{\max\{\nu_A(e), \nu_B(x) \}, \max\{\nu_A(e), \nu_B(y) \} \} = \max\{\nu_B(x), \nu_B(y)\}\leq \max\{\nu_B(x), \nu_B(y)\}$. Therefore, $\nu_B(x+y) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R_2. Also, $\nu_B(xy) = \max\{\nu_B(xy), \nu_A(ee)\} = \max\{\nu_A(ee), \nu_B(xy)\} = \nu_{AxB}((ee), (xy))= \nu_{AxB}[(e, x)(e, y)] \leq \max\{\nu_{AxB}(e, x), \nu_{AxB}(e, y) \} =\max\{\max\{\nu_A(e), \nu_B(x)\}, \max\{\nu_A(e), \nu_B(y)\}\}= \max\{\nu_B(x), \nu_B(y)\}$. Therefore, $\nu_B(xy) \leq \max\{\nu_B(x), \nu_B(y)\}$, for all x and y in R_2. Hence B is an intuitionistic fuzzy subhemiring of a hemiring R_2.Thus (ii) is proved.(iii) is clear.

2.6 Theorem

Let A be an intuitionistic fuzzy subset of a hemiring R and V be the strongest intuitionistic fuzzy relation of R. Then A is an intuitionistic fuzzy subhemiring of R if and only if V is an intuitionistic fuzzy subhemiring of RxR.

Proof: Suppose that A is an intuitionistic fuzzy subhemiring of a hemiring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR. We have, $\mu_V(x+y) = \mu_V [(x_1, x_2)+(y_1, y_2)] = \mu_V(x_1+ y_1 , x_2+ y_2) = \min\{\mu_A(x_1 + y_1), \mu_A(x_2+ y_2) \} \geq \min\{\min\{\mu_A(x_1), \mu_A(y_1) \}, \min\{\mu_A(x_2), \mu_A(y_2) \}\}= \min\{\min\{\mu_A(x_1), \mu_A(x_2) \}, \min\{\mu_A(y_1), \mu_A(y_2) \}\}= \min \{\mu_V (x_1, x_2), \mu_V (y_1, y_2) \}=\min\{\mu_V (x), \mu_V (y) \}$. Therefore, $\mu_V (x+y) \geq \min\{\mu_V (x), \mu_V (y)\}$, for all x and y in RxR. And, $\mu_V (xy) =\mu_V [(x_1, x_2)(y_1, y_2)]= \mu_V(x_1y_1 , x_2y_2) = \min\{\mu_A(x_1y_1), \mu_A(x_2y_2) \} \geq \min\{\min\{\mu_A(x_1), \mu_A(y_1) \}, \min\{\mu_A(x_2), \mu_A(y_2) \} \} = \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2) \} \}= \min\{\mu_V (x_1, x_2), \mu_V (y_1, y_2) \}= \min\{\mu_V (x), \mu_V (y)\}$. Therefore, $\mu_V (xy) \geq \min \{\mu_V (x), \mu_V (y)\}$, for all x and y in RxR. Also we have, $\nu_V (x+y) = \nu_V [(x_1, x_2) +(y_1, y_2)]= \nu_V(x_1+ y_1 , x_2+ y_2) = \max\{\nu_A(x_1+ y_1), \nu_A(x_2+ y_2) \} \leq \max\{\max\{\nu_A(x_1), \nu_A(y_1) \}, \max \{ \nu_A(x_2), \nu_A(y_2) \} \} = \max\{\max \{\nu_A(x_1), \nu_A(x_2) \}, \max\{\nu_A(y_1), \nu_A(y_2) \} \} = \max\{\nu_V (x_1, x_2), \nu_V (y_1, y_2) \}= \max\{\nu_V (x), \nu_V (y) \}$. Therefore, $\nu_V (x+y)\leq \max\{\nu_V (x), \nu_V (y) \}$, for all x and y in RxR. And, $\nu_V(xy) = \nu_V[(x_1, x_2)(y_1, y_2)]= \nu_V(x_1y_1 , x_2y_2)= \max\{ \nu_A(x_1y_1), \nu_A(x_2y_2) \} \leq \max \{ \max \{ \nu_A(x_1), \nu_A(y_1) \}, \max\{\nu_A(x_2), \nu_A(y_2) \} \} = \max\{\max\{\nu_A(x_1), \nu_A(x_2) \}, \max\{\nu_A(y_1), \nu_A(y_2) \} \}= \max\{\nu_V (x_1, x_2), \nu_V (y_1, y_2)\}= \max\{ \nu_V (x), \nu_V (y)\}$. Therefore, $\nu_V (xy) \leq \max \{\nu_V (x), \nu_V (y) \}$, for all x and y in RxR. This proves that V is an intuitionistic fuzzy subhemiring of RxR. Conversely assume that V is an intuitionistic fuzzy subhemiring of RxR, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in RxR, we have $\min\{ \mu_A(x_1+ y_1), \mu_A(x_2+ y_2) \}= \mu_V(x_1+y_1 , x_2+y_2) = \mu_V [(x_1, x_2) + (y_1, y_2)] = \mu_V(x+y) \geq \min\{\mu_V (x), \mu_V (y) \} = \min\{\mu_V (x_1, x_2) , \mu_V (y_1, y_2) \}= \min\{\min\{\mu_A(x_1), \mu_A(x_2) \}, \min\{\mu_A(y_1), \mu_A(y_2) \}\}$. If we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1+ y_1) \geq \min\{ \mu_A(x_1), \mu_A(y_1) \}$, for all x_1 and y_1 in R. And, $\min \{\mu_A(x_1y_1), \mu_A(x_2y_2) \} = \mu_V(x_1y_1 , x_2y_2) = \mu_V[(x_1, x_2) (y_1, y_2)]= \mu_V (xy) \geq \min\{\mu_V (x), \mu_V (y) \}= \min\{\mu_V$

(x_1, x_2), $\mu_V(y_1, y_2)$ }=min{min{$\mu_A(x_1)$, $\mu_A(x_2)$ }, min{$\mu_A(y_1)$, $\mu_A(y_2)$ } }. If we put $x_2 = y_2 = 0$, we get $\mu_A(x_1y_1) \geq$ min{$\mu_A(x_1)$, $\mu_A(y_1)$ }, for all x_1 and y_1 in R. Also we have, max{ $\nu_A(x_1+ y_1)$, $\nu_A(x_2+ y_2)$} = $\nu_V(x_1+ y_1 , x_2+y_2)$ = $\nu_V [(x_1, x_2) + (y_1, y_2)]$ = $\nu_V (x + y) \leq$ max{ $\nu_V (x)$, $\nu_V (y)$ }= max{$\nu_V (x_1, x_2)$, $\nu_V (y_1, y_2)$ }= max{max{$\nu_A(x_1)$, $\nu_A(x_2)$ }, max{$\nu_A(y_1)$, $\nu_A(y_2)$ } }. If we put $x_2 = y_2 = 0$, we get, $\nu_A(x_1+ y_1) \leq$ max{$\nu_A(x_1)$, $\nu_A(y_1)$ }, for all x_1 and y_1 in R. And, max{$\nu_A(x_1y_1)$, $\nu_A(x_2y_2)$ }= $\nu_V(x_1y_1 , x_2y_2)$ = $\nu_V[(x_1, x_2) (y_1, y_2)]$ = $\nu_V(xy) \leq$ max{$\nu_V (x)$, $\nu_V (y)$} = max{$\nu_V (x_1, x_2)$, $\nu_V (y_1, y_2)$ }= max { max {$\nu_A(x_1)$, $\nu_A(x_2)$ }, max{$\nu_A(y_1)$, $\nu_A(y_2)$ } }. If we put $x_2 = y_2 = 0$, we get, $\nu_A(x_1y_1) \leq$ max { $\nu_A(x_1)$, $\nu_A(y_1)$ }, for all x_1 and y_1 in R. Therefore, A is an intuitionistic fuzzy subhemiring of R.

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