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EQUILIBRIUM POINTS IN GENERALISED ELLIPTIC PHOTOGRAVITATIONAL RESTRICTED THREE BODY PROBLEMS WITH OBLATENESS

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Abstract
We have found the location of triangular equilibrium point in generalized elliptic photogravitational restricted three body problems. The problem is generalised in the sense that smaller primary is an oblate spheroid and bigger primary is radiating. The position of triangular point is affected by radiation oblateness, eccentricity and semi-major axis. The classical results may be verified.

Keywords: Equilibrium point, Photogravitational,Generalised, ERTBP.

1. Introduction

Elliptic orbit is more realistic than the circular. The elliptic restricted three body problem is a generalization of the classical RTBP. The elliptic restricted three body problem describes the three dimensional motion of a small particle, called the third body (infinitesimal mass) under the gravitational attraction force of two finite bodies, called the primaries, which revolve in elliptic orbit in a plane around their common centre of mass.

Radiation and oblateness of the primaries also affect the motion of the infinitesimal mass. Many researchers studied the restricted problem taking into account one or both the primaries as oblate spheroids and radiating. Radzievsky (1950) dealt with the restricted problem of three bodies, considering more massive primary as a source of radiation. Sahoo and Ishwar (2000) examined stability of collinear points in the generalized photo gravitational ERTB. A Narayan and C. R. Kumar (2011) studied the effect of photo gravitational and oblateness on the triangular Lagrangian points in ERTBP.

In section 2, we have found equations of motion of our problem. The mean motion of our problem is also found. It is affected by the eccentricity and semi major axis of orbit and oblateness of smaller primary.

In section 3, we have used perturbation method to find the location of triangular equilibrium points. We have found that they are the function of oblateness (A_2), radiation q_1 eccentricity and semi major axis of orbit. They are also different from classical case.

We conclude that the triangular equilibrium point is affected by radiation, oblateness, eccentricity and semi-major axis. The results have been verified by classical case.

2. Equations of Motion

We consider two bodies (primaries) of masses m_1 and m_2 with $m_1 > m_2$ moving in a plane around their common center of mass in elliptic orbit and a third body (infinitesimal mass) of mass m is moving in a plane of motion of the primaries. Equations of motion of our problem in rotating and pulsating co-ordinate system are given by (Sahoo and Ishwar 2000)

$$x'' - 2y' = \frac{\partial \Omega}{\partial x} = U_x \tag{1}$$

$$y'' - 2x' = \frac{\partial \Omega}{\partial y} = U_y \tag{2}$$

$$z'' = \frac{\partial \Omega}{\partial z} = U_z \tag{3}$$

Where the force $U = \Omega$

$$U = \frac{1}{\sqrt{1-e^2}}\left[\frac{x^2+y^2}{2} + \frac{1}{n^2}\left\{\frac{(1-«)q_1}{r_1} + \frac{«}{r_2} + \frac{« A_2}{2r_2^3}\right\}\right] \tag{4}$$

$$\Omega = \frac{1}{\sqrt{1-e^2}}\left[\frac{x^2+y^2}{2} + \frac{1}{n^2}\left\{\frac{(1-«)q_1}{r_1} + \frac{«}{r_2} + \frac{« A_2}{2r_2^3}\right\}\right] \tag{5}$$

The mean motion of our problem is obtained as

$$n^2 = \frac{\left(1+\frac{3A_2}{2}\right)\sqrt{1+e^2}}{a(1-e^2)} \tag{6}$$

a is semi-major axis of the ellipse

e is the eccentricity of the ellipse

A_2 is the coefficients of oblateness of smaller primary

$$r_i = (x+x_i)^2 + y^2 + z^2 \qquad i = 1,2 \tag{7}$$

$$x_1 = -«, x_2 = 1-«, « = \frac{m_2}{m_1+m_2} \tag{8}$$

Here m_1, m_2 are the masses of the bigger and smaller primaries (x_1,0,0) and (x_2,0,0) are the coordinate of m_1 and m_2 respectively. q_1 is mass reduction factor. $A_2 = \frac{r_e^2 - r_p^2}{5r^2}$ is oblateness coefficient due to smaller primary m_2, where r_e, r_p represents equatorial radii and polar radii respectively. $r_i(i=1,2)$ are the distance of the infinitesimal mass from m_1 and m_2 respectively. Semi-major axis and eccentricity of orbit is denoted by a and e respectively.

Now multiplying equations (1), (2) and (3) by $2x', 2y'$ and $2z'$ respectively and adding we get

$$\frac{dC}{dt} = -2(x'F_x + y'F_y) \tag{9}$$

where $C = 2U - x'^2 - y'^2$, the quantity C is Jacobi integral. The zero velocity curves are given by $C = 2U$.

3. Locations of the Equilibrium Points

Using perturbation method, we have found location of triangular equilibrium point. For triangular equilibrium point $U_x = 0, U_y = 0, y \neq 0$ and $z = 0$ then we have

$$x - \frac{1}{n^2}\left\{\frac{(1-«)(x+«)q_1}{r_1^3} + \frac{«(x+«-1)}{r_2^3} + \frac{3«A_2(x+«-1)}{2r_2^5}\right\} = 0 \quad (10)$$

$$y - \frac{1}{n^2}\left\{\frac{(1-«)yq_1}{r_1^3} + \frac{«\, y}{r_2^3} + \frac{3«A_2 y}{2r_2^5}\right\} = 0 \quad (11)$$

$$\left\{\frac{(1-«)zq_1}{r_1^3} + \frac{«\, z}{r_2^3} + \frac{3«A_2 z}{2r_2^5}\right\} = 0 \quad (12)$$

Now, from equation (10), (11) and (12), we have

$$x - \frac{1}{n^2}\left\{\frac{(1-«)(x+«)q_1}{r_1^3} + \frac{«(x+«-1)}{r_2^3} + \frac{3«A_2(x+«-1)}{2r_2^5}\right\} = 0 \quad (13)$$

$$y - \frac{1}{n^2}\left\{\frac{(1-«)yq_1}{r_1^3} + \frac{«\, y}{r_2^3} + \frac{3«A_2 y}{2r_2^5}\right\} = 0 \quad (14)$$

$$\left\{n^2 - \frac{(1-«)q_1}{r_1^3} - \frac{«}{r_2^3} - \frac{3«A_2}{2r_2^5}\right\} y_0 = 0 \quad (15)$$

where

$$y_0 = \pm\left[¢^2(1-e^2) - \frac{1}{4}\left\{1 + 2(¢^2 - a^{2/3})(1-e^2)\right\}\right]$$

$$¢ = (aq_1)^{1/3}$$

Multiplying equations (13) and (14) by y and $(x+«)$ respectively and subtracting

$$-y« + \frac{y«}{n^2 r_2^3} + \frac{3«A_2 y}{2n^2 r_2^5} = 0$$

$$-y«\left[1 - \frac{1}{n^2}\left\{\frac{1}{r_2^3} + \frac{3A_2}{2r_2^5}\right\}\right] = 0 \quad (16)$$

$$\left[n^2 - \frac{1}{r_2^3} - \frac{3A_2}{2r_2^5}\right]«\, y_0 = 0 \quad (17)$$

In photogravitational ERTBP i.e., when oblateness is absent and bigger primary is radiating then

$$r_1 = \left(\frac{q_1}{n^2}\right)^{1/3} \quad (18)$$

$$r_2 = \frac{1}{n^{2/3}} \quad (19)$$

Now we suppose due to oblateness perturbation in r_2 is $\in_2$ i.e.,

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$$r_2 = \frac{1}{n^{2/3}} + \in_2 \tag{20}$$

Again from the relation between mean motion and oblateness, we have

$$n^2 = \frac{\left(1 + \frac{3A_2}{2}\right)\sqrt{1+e^2}}{a\left(1-e^2\right)} \tag{21}$$

If $A_2 = 0$ in equation (21), we get

$$n^2 = \frac{\sqrt{1+e^2}}{a\left(1-e^2\right)} = \frac{1}{a}\left(1+e^2\right)^{1/2}\left(1+e^2\right)^{-1} = \frac{1}{a}\left(1+e^2+\frac{e^2}{2}+\frac{e^4}{4}\right)$$

$$n^2 = \frac{1}{a}\left(1+\frac{3e^2}{2}\right) \text{(neglecting higher order terms)} \tag{22}$$

With the help of equation (22), we have

$$\frac{1}{n^{2/3}} = a^{1/3}\left(1-\frac{e^2}{2}\right) \tag{23}$$

With the help of equation (23), we get from equation (20)

$$r_2 = a^{1/3}\left(1-\frac{e^2}{2}\right) + \in_2 \tag{24}$$

$$\frac{1}{r_2} = \frac{1}{a^{1/3}\left(1-\frac{e^2}{2}\right) + \in_2}$$

$$\frac{1}{r_2^3} = \frac{1}{a}\left(1-\frac{e^2}{2}\right)^{-3}\left\{1 + \in_2 a^{-1/3}\left(1+\frac{e^2}{2}\right)\right\}^{-3}$$

$$\frac{1}{r_2^3} = \frac{1}{a}\left(1+\frac{3e^2}{2} - 3\in_2 a^{-1/3} - 6\in_2 e^2 a^{-1/3}\right) \tag{25}$$

(neglecting higher order terms)

In similar way, we find

$$\frac{1}{r_2^5} = \frac{1}{a^{5/3}}\left(1+\frac{5e^2}{2} - 5\in_2 a^{-1/3} - 15\in_2 e^2 a^{-1/3}\right) \tag{26}$$

Considering only terms e^2 and A_2 and neglecting their product, equation (21) gives

$$n^2 = \frac{1}{a}\left(1+\frac{3A_2}{2}\right)\left(1+\frac{3e^2}{2}\right)$$

$$n^2 = \frac{1}{a}\left(1+\frac{3A_2}{2}+\frac{3e^2}{2}\right) \tag{27}$$

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Now substituting the values of $\frac{1}{r_2^3}, \frac{1}{r_2^5}, n^2$ from equations (25), (26) and (27) in equation (17), we obtain

$$\frac{1}{a}\left(1+\frac{3A_2}{2}+\frac{3e^2}{2}\right)-\frac{1}{a}\left(1+\frac{3e^2}{2}-3\in_2 a^{-1/3}-6\in_2 e^2a^{-1/3}\right)$$
$$-\frac{3A_2}{2a^{5/3}}\left(1+\frac{5e^2}{2}-5\in_2 a^{-1/3}-15\in_2 e^2a^{-1/3}\right)=0$$

$$\Bigg[1+\frac{3A_2}{2}+\frac{3e^2}{2}-1-\frac{3e^2}{2}+3\in_2 a^{-1/3}+6\in_2 e^2a^{-1/3}-\frac{3A_2}{2a^{2/3}}$$
$$-\frac{15A_2e^2}{4a^{2/3}}+\frac{15}{2}A_2\in_2 a^{-1}+\frac{45}{2}A_2e^2\in_2 a^{-1}\Bigg]=0$$

$$\in_2=\frac{-\frac{1}{3a^{-1/3}}\left(\frac{3A_2}{2}-\frac{3}{2}A_2a^{-2/3}-\frac{15}{4}A_2e^2a^{-2/3}\right)}{\left(1+2e^2+\frac{5}{2}A_2a^{-2/3}+\frac{15}{2}A_2e^2a^{-2/3}\right)}$$

$$\in_2=\left[-\frac{1}{3a^{-1/3}}\left(\frac{3A_2}{2}-\frac{3}{2}A_2a^{-2/3}-\frac{15}{4}A_2e^2a^{-2/3}\right)\right]$$
$$\left(1-2e^2-\frac{5}{2}A_2a^{-2/3}-\frac{15}{2}A_2e^2a^{-2/3}\right)$$

$$\in_2=-\frac{1}{3a^{-1/3}}\Bigg(\frac{3A_2}{2}-\frac{3}{2}A_2a^{-2/3}-\frac{15}{4}A_2e^2a^{-2/3}$$
$$-3A_2e^2+3A_2e^2a^{-2/3}+\frac{15}{2}A_2e^4a^{-2/3}\Bigg)$$

Neglecting higher order terms and product of A_2e^2, we have

$$\in_2=-\frac{1}{2}A_2a^{1/3}\left(1-a^{-2/3}\right) \tag{28}$$

Now from equation (27)

$$n=\frac{1}{a^{1/2}}\left(1+\frac{3A_2}{4}+\frac{3e^2}{4}\right)$$

From equation (24)

$$r_2=a^{1/3}\left(1-\frac{e^2}{2}\right)-\frac{1}{2}A_2a^{1/3}\left(1-a^{-2/3}\right) \tag{29}$$

$\frac{1}{r_1} = \frac{\frac{1}{q_1^{1/3}}}{n^{2/3}}$ and substituting value of $\frac{1}{n^{2/3}}$ from equation (23) in equation (18) and neglecting product of $A_2 e^2$, we get

$$r_1 = (aq_1)^{1/3}\left(1 - \frac{e^2}{2}\right) - \frac{A_2 (aq_1)^{1/3}(1 - 2e^2)}{2}$$

$$r_1 = (aq_1)^{1/3}\left(1 - \frac{e^2}{2}\right) - \frac{1}{2} A_2 (aq_1)^{1/3} \quad (30)$$

Now

$$r_1^2 = \left[(aq_1)^{1/3}\left(1 - \frac{e^2}{2}\right) - \frac{1}{2} A_2 (aq_1)^{1/3} \right]^2$$

Considering only first order terms, we get

$$r_1^2 = (aq_1)^{2/3}(1 - e^2) - (aq_1)^{2/3} A_2 \left(1 - \frac{5e^2}{2}\right) \quad (31)$$

Now from equation (29), we get

$$r_2^2 = \left[a^{1/3}\left(1 - \frac{e^2}{2}\right) - \frac{1}{2} A_2 a^{1/3}\left(1 - a^{-2/3}\right)\right]^2$$

Considering only first order terms, we have

$$r_2^2 = a^{2/3}\left[(1 - e^2) - A_2\left(1 - a^{-2/3}\right)\right] \quad (32)$$

Now, we have

$$x + « = \frac{r_1^2 - r_2^2 + 1}{2}$$

$$= \frac{r_1^2}{2} - \frac{r_2^2}{2} + \frac{1}{2}$$

Putting value of r_1^2 and r_2^2 from (31) and (32)

$$x + « = (aq_1)^{2/3}\frac{(1 - e^2)}{2} - (aq_1)^{2/3}\frac{A_2}{2} - a^{2/3}\frac{(1 - e^2)}{2} + \frac{A_2 a^{2/3}}{2}\left(1 - a^{-2/3}\right) + \frac{1}{2}$$

$$x = \frac{1}{2} - « + (aq_1)^{2/3}\frac{(1 - e^2)}{2} - a^{2/3}\frac{(1 - e^2)}{2} + \frac{A_2 a^{2/3}}{2}\left(1 - a^{-2/3}\right) - (aq_1)^{2/3}\frac{A_2}{2}$$

Putting $n = a^{-1/2}\left(1 + \frac{3e^2}{4} + \frac{3A_2}{4}\right)$ and neglecting higher order terms and product of $A_2 e^2$ we have

$$x = \frac{1}{2} - « + \frac{1}{2}\left[(aq_1)^{2/3}\left(1 - A_2 - e^2\right) - a^{2/3}\left(1 - A_2 - e^2 + A_2 a^{-2/3}\right)\right] \quad (33)$$

and $y^2 = r_1^2 - (x + «)^2 \quad (34)$

At first find $(x+«)^2$

$$(x+«)^2=\left[\frac{1}{2}+(aq_1)^{2/3}\frac{(1-e^2)}{2}-a^{2/3}\frac{(1-e^2)}{2}+\frac{A_2a^{2/3}}{2}\left(1-a^{-2/3}\right)-(aq_1)^{2/3}\frac{A_2}{2}\right]$$

$$\begin{aligned}(x+«)^2&=\frac{1}{4}+\frac{1}{4}\left(1-2e^2\right)\left((aq_1)^{2/3}-a^{2/3}\right)^2+\frac{\left((aq_1)^{2/3}-a^{2/3}\right)}{2}\left(1-e^2\right)\\&\quad-\frac{1}{2}A_2\left((aq_1)^{2/3}-a^{2/3}\right)\left(1+(aq_1)^{2/3}-a^{2/3}\right)-\frac{1}{2}A_2\left(1+(aq_1)^{2/3}-a^{2/3}\right)\end{aligned} \tag{35}$$

With the help of (31) and (34), we get

$$\begin{aligned}y^2&=(aq_1)^{2/3}\left(1-e^2\right)-(aq_1)^{2/3}A_2\left(1-\frac{5e^2}{2}\right)-\frac{1}{4}-\frac{1}{4}\left(1-2e^2\right)\left((aq_1)^{2/3}-a^{2/3}\right)^2\\&\quad-\frac{\left((aq_1)^{2/3}-a^{2/3}\right)}{2}\left(1-e^2\right)+\frac{1}{2}A_2\left((aq_1)^{2/3}-a^{2/3}\right)\left(1+(aq_1)^{2/3}-a^{2/3}\right)\\&\quad+\frac{1}{2}A_2\left(1+(aq_1)^{2/3}-a^{2/3}\right)\end{aligned}$$

$$\begin{aligned}y=\pm\Big[&(aq_1)^{2/3}\left(1-A_2-e^2\right)-\frac{1}{4}\Big\{1+2\left((aq_1)^{2/3}-a^{2/3}\right)\left(1-e^2\right)\\&+\left((aq_1)^{2/3}-a^{2/3}\right)^2\left(1-2e^2\right)-2A_2\left(1+(aq_1)^{2/3}-a^{2/3}\right)^2\Big\}\Big]\end{aligned} \tag{36}$$

4. Conclusion

We conclude that triangular equilibrium point of the problem is affected by radiation, oblateness, eccentricity and semi-major axis. We verified the results of classical case.

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