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A MARKOVIAN QUEUE WITH BLOCKING AND CATASTROPHES

M. Balasubramanian*   Dr. S. Bharathidass*   Dr. V. Ganesan**
**Assistant Professor, PG & Research Department of Statistics, Periyar E.V.R. College, Tiruchirappalli*
***Associate Professor (Retd), PG & Research Department of Statistics, Periyar E.V.R. College, Tiruchirappalli.*

Abstract
A queueing system with two servers is considered. Blocking and catastrophe are employed in this system. Arrival times of units and service times corresponding to both servers follow Markovian nature. The expressions for the expected queue length in the system and loss probability of units are derived. The numerical results are provided.

Key Words: Markovian Queue, Heterogeneous Servers, Blocking, Catastrophe, Loss Probability.

1. Introduction
Queueing systems are analysed by adopting one or more concepts like bulk size rule, vacation policy, heterogeneous arrival, heterogeneous service, balking, blocking and so on. There are many queueing models have been administered by two servers in which one server provides compulsory service and other server provides optional service. The heterogeneous services are performed due to the requirements of the units and the service mentality of the servers. There are different types of heterogeneous services.

- A single server has provided different services through a single window. (In a single man post office, he sells stamps, receives money orders, registered letters and so on).
- A single server has provided two stages of service such as,
  a. The first service is compulsory for all units and the second service is optional.
  b. Both services are compulsory for all units. (Billing the goods and delivery the goods).
  c. Units choose any one of the two counters and get service.
- Two servers have provided two kinds of services. One provides service to the allotted counter and also provides service in the neighbour's counter if he is free and his neighbor is on vacation.

In real systems, a catastrophes might result either from outside the system/facility or from another service station. Computer networks with a virus infection might be considered, for example, queueing models in series with catastrophes. Furthermore, the return of the system to the initial state either automatically or by the server due to the busy condition of the internet systems.

In this paper, a queueing system in series with two heterogeneous servers and the occurrence of catastrophe has been considered. The mathematical expressions for the expected number of units in the system and probability of lost units are derived. The results are reduced when the absence of catastrophe. Numerical values and curves are exhibited.

2. Review of Literature
A queueing model in series with blocking and no waiting space have been discussed by various researchers. **Altiok (1989)** has derived approximate solutions for a queue in series with blocking. **Alpaslam (1996)** has obtained probability of lost units of a queueing model in series. **Modiano et al (1996)** have studied queues in series on computer communication systems. **Madan (2000)** has introduced the concept of heterogeneous services in two stages for the M/G/1 model and derived mathematical expressions for queueing performances.

Ganesan (2001) has analysed a general bulk service Markovian queue in which the server provides two different services such as the first service is compulsory and the second is optional. The probability generating functions, average queue length and average waiting times in the first and second services have been obtained. Similarly, **Madan and Baklizi (2002)** have studied Poisson arrival queue with two stage heterogeneous services.

Krishnakumar et al (2007) have discussed two parallel servers Markovian model with the occurrence of catastrophes which follow Poisson distribution. **Sundar rajan and Ganesan (2010)** have studied a single server bulk arrival queue and each unit undergoes two stages of heterogeneous services. **Seo and Lee (2011)** have derived remarkable results for queue in series with blocking on wireless networks. **Isguder and Celikoglu (2012)** have discussed the minimization of loss probability. Also, **Isguder and Kaya (201**2) have studied the steady-state probabilities and loss probability for the series queues with catastrophes and heterogeneous servers.

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3. Formulation of the Model

Consider a queueing system with blocking and no waiting line subject to catastrophes. The arrival times of units to the two counters are distributed according to Poisson distribution with parameters a_1 and a_2 respectively. There are two heterogeneous servers in series in the system. Their mean service times are assumed to be different from each other. The service time of each unit at server 'k' is random variable $¦_k$ and has an exponential distribution with parameter $«_k$ (k = 1, 2).

$$P(¦_k \le t) = 1 - e^{-«_k t},\ t > 0 \qquad (1)$$

Apart from arrival and service processes, the catastrophes occur in a Poisson process with rate ¥ in the system. Let the instants of catastrophe be $C_{0,}$ C_1, and C2 . . .

Where $0 < C_0 < C_1 < . . .$ and $T = C_i – C_{i-1}$ for $i \geq 1$. Due to the relationship between Poisson distribution and exponential distribution, the time between two instants of catastrophe is exponentially distributed with parameter ¥ .

$$P(T \le t) = 1 - e^{-¥t},\ t > 0 \qquad (2)$$

As soon as the occurrence of a catastrophe in the system, all units are immediately destroyed. Both servers are inactivated momentarily and when there is a new arrival, both servers get ready to serve. In short, when there is a catastrophe in the system, the system returns to its initial state with probability one.

Each unit arriving in the system is first served at the first server and then at the second server. Waiting line is not allowed in front of the servers. If the second server is busy when the service time has been completed at the first server, then the first server is blocked until the service is completed at the second server. If the first server is busy or blocked at the time of arrival of a unit in the system, that the unit leaves the system without being served. Such units are called "lost units". Hence, the main problem of this study is to compute the probability of lost units in the system and minimize this probability.

4. Differential – Difference Equations of the Model

According to the above assumptions, the required equations are set out here,

$$\frac{dP_{00}(t)}{dt} = -(\lambda_1+\lambda_2)P_{00}(t)+\mu_2 P_{01}(t)+\gamma[1-P_{00}(t)] \qquad (3)$$

$$\frac{dP_{10}(t)}{dt} = -(\lambda_2+\mu+\gamma)P_{10}(t)+\lambda_1 P_{00}(t)+\mu_2 P_{11}(t) \qquad (4)$$

$$\frac{dP_{01}(t)}{dt} = -(\lambda_1+\mu_2+\gamma)P_{01}(t)+\mu_1 P_{10}(t)+\lambda_2 P_{10}(t)+\mu_2 P_{b1}(t) \qquad (5)$$

$$\frac{dP_{11}(t)}{dt} = -(\mu_1+\mu_2+\gamma)P_{11}(t)+\lambda_1 P_{01}(t)+\lambda_2 P_{10}(t) \qquad (6)$$

$$\frac{dP_{b1}(t)}{dt} = -(\mu_2+\gamma)P_{b1}(t)+\mu_1 P_{11}(t) \qquad (7)$$

Taking the limit as $t \to \infty$, the equations (3) to (7) reduce to

$$-(ª_1+ ª_2)P_{00}+ «_2P_{01}+ ¥ - ¥ P_{00} = 0 \quad (8)$$

$$-(ª_2 + « +¥)P_{10}+ª_1P_{00}+«_2P_{11} = 0 \quad (9)$$

$$-(ª_1 + «_2 +¥)P_{01}+«_1P_{10} +ª_2P_{10}+«_2P_{b1} = 0 \quad (10)$$

$$-(«_1 + «_2 +¥)P_{11}+ª_1P_{01} +ª_2P_{10} = 0 \quad (11)$$

$$-(«_2 +¥)P_{b1}+«_1P_{11} = 0 \quad (12)$$

5. Mean Queue Size and Loss Probability

On solving the steady-state equations from (8) to (12), the required steady-state probabilities, P_{ij} (i = 0,1,b; j = 0,1) in terms of P_{00} are obtained as under,

$$P_{01}= \frac{(ª_1+ª_2+¥)P_{00}-¥}{«_2} \quad (13)$$

$$P_{10}= \frac{ª_1(ª_1+ª_2+«_1+«_2+2¥)P_{00}-ª_1¥}{(«_1+¥)(ª_2+«_1+«_2+¥)} \quad (14)$$

$$P_{b1}=[\{(ª_1+«_2+¥)(ª_1+ª_2+¥)(«_1+¥)(ª_2+«_1+«_2+¥)$$

$$-ª_1«_2(«_1+ª_2)(ª_1+ª_2+«_1+«_2+2¥)\}P_{00}$$

$$+ª_1ª_2(«_1+ª_2)¥-(ª_1+«_2+¥)(«_1+¥)(ª_2+«_1+«_2+¥)]$$

$$[«_2^2(«_1+¥)(ª_2+«_1+«_2+¥)]^{-1} \quad (15)$$

and

$$P_{11}= \left(\frac{«_1+«_2+¥}{«_1}\right)P_{b1} \quad (16)$$

In this stage, there is a need to estimate P_{00} . For this purpose, use the normalizing condition

$$P_{00} + P_{01}+ P_{11}+ P_{b1} = 1$$ Which yields

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$$P_{00} = \Big[\mu_1\mu_2(\mu_1+\gamma)(\mu_2+\gamma)(\lambda_2+\mu_1+\mu_2+\gamma)+\lambda_1\mu_1\mu_2^2\gamma$$

$$-\ \{\lambda_1\lambda_2(\mu_1+\lambda_2)\gamma-(\lambda_1+\mu_2+\gamma)(\mu_1+\gamma)(\lambda_2+\mu_1+\mu_2+\gamma)\}\ (\mu_1+\mu_2+\gamma)\Big]$$

$$\Big[(\mu_1+\gamma)(\lambda_2+\mu_1+\mu_2+\gamma)\{\mu_1\mu_2(\lambda_1+\lambda_2+\mu_2+\gamma)+(\lambda_1+\mu_2+\gamma)(\lambda_1+\lambda_2+\gamma)(\mu_1+\mu_2+\gamma)\}$$

$$-\lambda_1\mu_2(\lambda_1+\lambda_2+\mu_1+\mu_2+2\gamma)\{\mu_1(\mu_1+\lambda_2+\gamma)+\lambda_2(\mu_1+\gamma)\}\ \Big]^{-1} \qquad (17)$$

The expected queue size in the system (L_S) is derived by using the equations from (13) to (16),

$$L_s = P_{01} + P_{10} + 2(P_{11}+P_{b1})$$

$$= 1 + \Big[\{(\mu_1+\gamma)(\lambda_2+\mu_1+\mu_2+\gamma)\ \{(\mu_1+\mu_2+\gamma)(\lambda_1+\mu_2+\gamma)(\lambda_1+\lambda_2+\gamma)-\mu_1\mu_2^3\}$$

$$-\lambda_1\mu_2(\mu_1+\lambda_2)(\mu_1+\mu_2+\gamma)(\lambda_1+\lambda_2+\mu_1+\mu_2+2\gamma)\}P_{00}$$

$$+\lambda_1\lambda_2(\mu_1+\lambda_2)(\mu_1+\mu_2+\gamma)\gamma-(\lambda_1+\mu_2+\gamma)(\mu_1+\gamma)(\mu_1+\mu_2+\gamma)(\lambda_2+\mu_1+\mu_2+\gamma)\Big]$$

$$\Big[\mu_1\mu_2^2(\mu_1+\gamma)(\lambda_2+\mu_1+\mu_2+\gamma)\Big]^{-1} \qquad (18)$$

The loss probability of units is obtained as

$$P_L = P_{10} + P_{11} + P_{b1})$$

$$= \frac{1}{«_2}\Big[«_2 + ¥ - («_2 + ª_1 + ª_2 + ¥)P_{00}\Big] \qquad (19)$$

6. Particular Case

By considering the absence of the catastrophe $(¥ = 0)$ in the system, the expected queue size in the system (18) and loss probability of units (19) are reduced as,

$$L_s = 1 + \Big[\{\mu_1(\lambda_2+\mu_1+\mu_2)\ \{(\mu_1+\mu_2)(\lambda_1+\mu_2)(\lambda_1+\lambda_2)-\mu_1\mu_2^2\}$$

$$-\lambda_1\mu_2(\mu_1+\lambda_2)(\mu_1+\mu_2)(\lambda_1+\lambda_2+\mu_1+\mu_2)\}P_{00}$$

$$-\mu_1(\lambda_1+\mu_2)(\mu_1+\mu_2)(\lambda_2+\mu_1+\mu_2)\Big]\ \Big[\mu_1^2\mu_2^2(\mu_1+\gamma)(\lambda_2+\mu_1+\mu_2)\Big]^{-1} \qquad (20)$$

and

$$P_L = \frac{1}{«_2}\Big[«_2 - («_2 + ª_1 + ª_2)P_{00}\Big] \qquad (21)$$

where

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$$P_{00} = \left[«_1^2 «_2^2 (ª_2 + «_1 + «_2) + (ª_1 + «_2) «_1 (ª_1 + «_1 + «_2)(«_1 + «_2) \right]$$

$$\left[«_1 (ª_2 + «_1 + «_2) \left\{ «_1 «_2 (ª_1 + ª_2 + «_2) + (ª_1 + «_2)(ª_1 + ª_2)(«_1 + «_2) \right\} \right.$$

$$\left. - ª_1 «_2 (ª_1 + ª_2 + «_1 + «_2) \left\{ «_1 («_1 + «_2) + ª_2 «_1 \right\} \right]^{-1}$$

The reduced expected queue size (20) and loss probability (21) coincide with the results of Alpaslan (1996).

7. Numerical Illustrations

The derived mathematical expressions given in equations from (18) to (21) are to be justified by assuming different values of the specific parameters. The values of the parameters are considered as $ª_1 = 1$ to 10, $ª_2 = 4$, $(«_1, «_2)$ = (3,4), (4,3), (4,5), (5,4), (5,6), (6,5) *and* $¥ = 0,\ 0.3,\ 0.4,\ 0.5$.

The expected queue size in the system (L_S) and loss probability of units (P_L) are computed by using the above values and presented in the following tables. The nature of these values are exhibited.

Table No: 1

$ª_1$ \ $«_1$, $«_2$	L_s ($ª_2 = 4$; $¥ = 0$)						P_L ($ª_2 = 4$; $¥ = 0$)					
$«_1$	**3**	**4**	**4**	**5**	**5**	**6**	**3**	**4**	**4**	**5**	**5**	**6**
$«_2$	**4**	**3**	**5**	**4**	**6**	**5**	**4**	**3**	**5**	**4**	**6**	**5**
1	*0.6696*	*0.9419*	*0.6849*	*0.8896*	*0.6866*	*0.8571*	*0.1914*	*0.1214*	*0.2307*	*0.1862*	*0.2473*	*0.2161*
2	*0.5933*	*1.0013*	*0.6307*	*0.8949*	*0.6502*	*0.8506*	*0.1333*	*0.0644*	*0.2012*	*0.1556*	*0.2328*	*0.1999*
3	*0.5330*	*1.0870*	*0.5914*	*0.9184*	*0.6258*	*0.8579*	*0.0920*	*0.0302*	*0.1834*	*0.1399*	*0.2270*	*0.1945*
4	*0.4884*	*1.1917*	*0.5647*	*0.9552*	*0.6112*	*0.8753*	*0.0640*	*0.0121*	*0.1743*	*0.1346*	*0.2274*	*0.1965*
5	*0.4581*	*1.3095*	*0.5483*	*1.0012*	*0.6041*	*0.8997*	*0.0462*	*0.0047*	*0.1715*	*0.1363*	*0.2321*	*0.2035*
6	*0.4401*	*1.4361*	*0.5401*	*1.0536*	*0.6028*	*0.9289*	*0.0358*	*0.0042*	*0.1731*	*0.1425*	*0.2397*	*0.2136*
7	*0.4324*	*1.5682*	*0.5384*	*1.1100*	*0.6057*	*0.9611*	*0.0307*	*0.0081*	*0.1777*	*0.1513*	*0.2491*	*0.2255*
8	*0.4331*	*1.7040*	*0.5416*	*1.1691*	*0.6118*	*0.9953*	*0.0295*	*0.0145*	*0.1843*	*0.1617*	*0.2596*	*0.2383*
9	*0.4406*	*1.8421*	*0.5487*	*1.2298*	*0.6202*	*1.0305*	*0.0309*	*0.0225*	*0.1921*	*0.1728*	*0.2707*	*0.2514*
10	*0.4538*	*1.9815*	*0.5587*	*1.2913*	*0.6303*	*1.0663*	*0.0342*	*0.0312*	*0.2006*	*0.1841*	*0.2819*	*0.2644*

Figure No.1.1

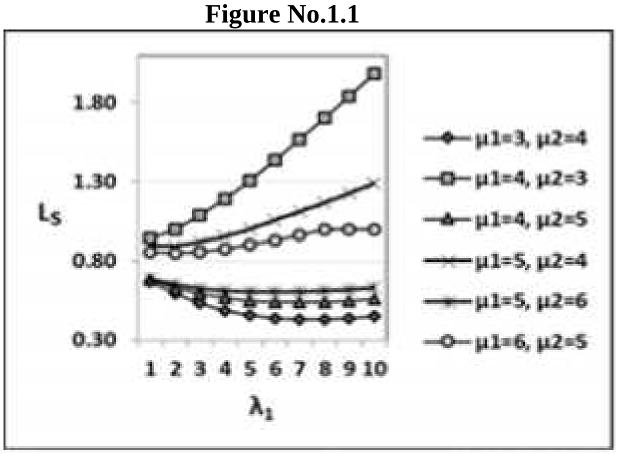

Figure No.1.2

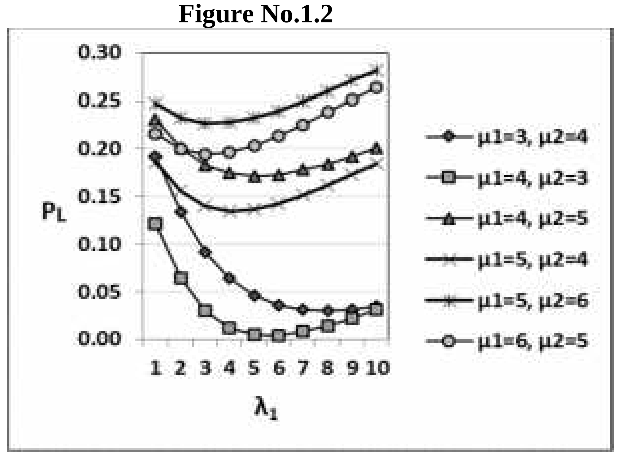

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Table No: 2

a1 \ «1 / «2	L_s (a$_2$ = 4; ¥ = 0.3)						P_L (a$_2$ = 4; ¥ = 0.3)					
«1	**3**	**4**	**4**	**5**	**5**	**6**	**3**	**4**	**4**	**5**	**5**	**6**
«2	**4**	**3**	**5**	**4**	**6**	**5**	**4**	**3**	**5**	**4**	**6**	**5**
1	*0.7883*	*1.0360*	*0.7816*	*0.9734*	*0.7688*	*0.9309*	*0.2983*	*0.2424*	*0.3093*	*0.2726*	*0.3103*	*0.2839*
2	*0.7483*	*1.0971*	*0.7484*	*0.9871*	*0.7463*	*0.9326*	*0.2727*	*0.2195*	*0.2971*	*0.2605*	*0.3064*	*0.2792*
3	*0.7250*	*1.1770*	*0.7285*	*1.0151*	*0.7341*	*0.9456*	*0.2596*	*0.2127*	*0.2939*	*0.2598*	*0.3094*	*0.2831*
4	*0.7157*	*1.2689*	*0.7189*	*1.0526*	*0.7298*	*0.9663*	*0.2554*	*0.2157*	*0.2970*	*0.2665*	*0.3170*	*0.2926*
5	*0.7174*	*1.3679*	*0.7170*	*1.0961*	*0.7313*	*0.9919*	*0.2571*	*0.2246*	*0.3041*	*0.2775*	*0.3276*	*0.3054*
6	*0.7277*	*1.4709*	*0.7210*	*1.1434*	*0.7368*	*1.0207*	*0.2625*	*0.2365*	*0.3138*	*0.2908*	*0.3400*	*0.3201*
7	*0.7446*	*1.5761*	*0.7291*	*1.1927*	*0.7452*	*1.0512*	*0.2703*	*0.2499*	*0.3249*	*0.3053*	*0.3533*	*0.3355*
8	*0.7666*	*1.6822*	*0.7404*	*1.2429*	*0.7556*	*1.0825*	*0.2795*	*0.2637*	*0.3367*	*0.3201*	*0.3669*	*0.3512*
9	*0.7924*	*1.7884*	*0.7539*	*1.2935*	*0.7673*	*1.1141*	*0.2894*	*0.2774*	*0.3488*	*0.3346*	*0.3804*	*0.3665*
10	*0.8211*	*1.8943*	*0.7690*	*1.3440*	*0.7798*	*1.1456*	*0.2994*	*0.2906*	*0.3607*	*0.3487*	*0.3936*	*0.3813*

Figure No.2.1

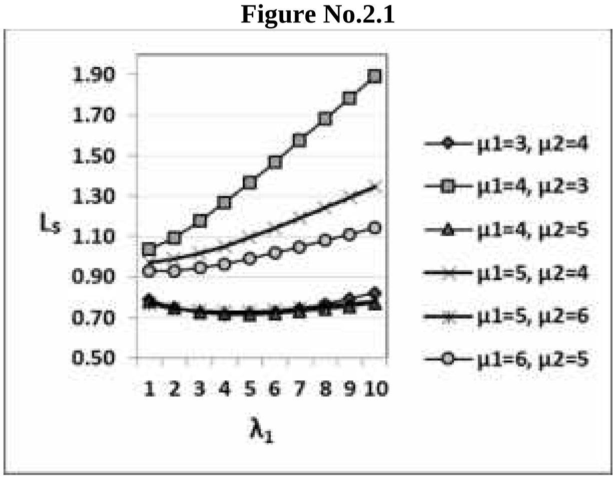

Figure No.2.2

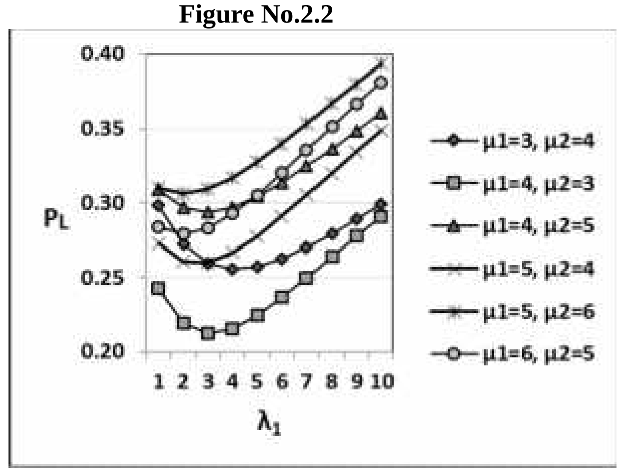

Table No. 3

a1 \ «1 / «2	L_s (a$_2$ = 4; ¥ = 0.4)						P_L (a$_2$ = 4; ¥ = 0.4)					
«1	**3**	**4**	**4**	**5**	**5**	**6**	**3**	**4**	**4**	**5**	**5**	**6**
«2	**4**	**3**	**5**	**4**	**6**	**5**	**4**	**3**	**5**	**4**	**6**	**5**
1	*0.8256*	*1.0671*	*0.8128*	*1.0013*	*0.7957*	*0.9555*	*0.3326*	*0.2820*	*0.3351*	*0.3012*	*0.3312*	*0.3066*
2	*0.7929*	*1.1264*	*0.7848*	*1.0166*	*0.7769*	*0.9593*	*0.3154*	*0.2682*	*0.3276*	*0.2943*	*0.3303*	*0.3051*
3	*0.7763*	*1.2023*	*0.7692*	*1.0449*	*0.7678*	*0.9735*	*0.3094*	*0.2685*	*0.3284*	*0.2978*	*0.3357*	*0.3117*
4	*0.7725*	*1.2881*	*0.7631*	*1.0816*	*0.7659*	*0.9946*	*0.3108*	*0.2770*	*0.3346*	*0.3077*	*0.3454*	*0.3233*
5	*0.7787*	*1.3795*	*0.7640*	*1.1235*	*0.7692*	*1.0202*	*0.3171*	*0.2899*	*0.3444*	*0.3212*	*0.3576*	*0.3377*
6	*0.7923*	*1.4737*	*0.7700*	*1.1683*	*0.7762*	*1.0484*	*0.3262*	*0.3049*	*0.3561*	*0.3365*	*0.3712*	*0.3536*
7	*0.8115*	*1.5691*	*0.7797*	*1.2146*	*0.7856*	*1.0779*	*0.3370*	*0.3207*	*0.3689*	*0.3524*	*0.3855*	*0.3701*
8	*0.8349*	*1.6647*	*0.7918*	*1.2615*	*0.7967*	*1.1080*	*0.3485*	*0.3363*	*0.3820*	*0.3683*	*0.3999*	*0.3865*
9	*0.8613*	*1.7600*	*0.8058*	*1.3083*	*0.8087*	*1.1381*	*0.3602*	*0.3514*	*0.3951*	*0.3838*	*0.4141*	*0.4024*
10	*0.8900*	*1.8546*	*0.8209*	*1.3548*	*0.8214*	*1.1679*	*0.3718*	*0.3658*	*0.4078*	*0.3986*	*0.4279*	*0.4177*

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Figure No.3.1

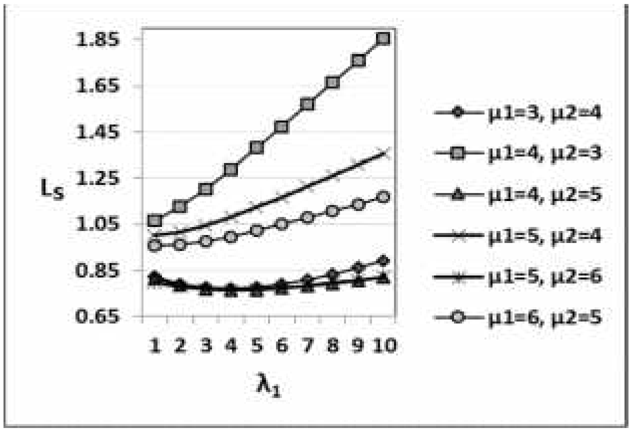


Figure No.3.2

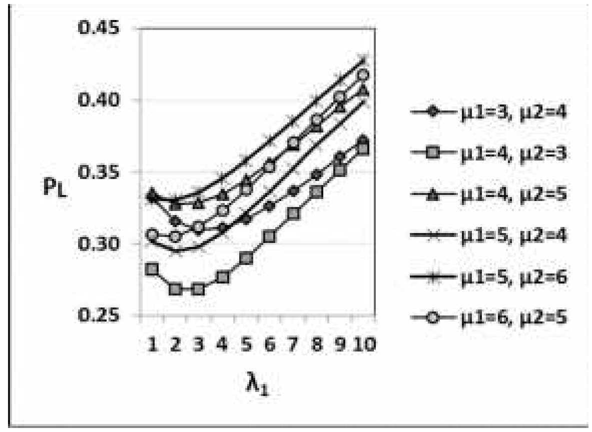


Table No. 4

a1 \ «1 / «2	L_s (a$_2$ = 4; ¥ = 0.5)						P_L (a$_2$ = 4; ¥ = 0.5)					
«1	**3**	**4**	**4**	**5**	**5**	**6**	**3**	**4**	**4**	**5**	**5**	**6**
«2	**4**	**3**	**5**	**4**	**6**	**5**	**4**	**3**	**5**	**4**	**6**	**5**
1	*0.8620*	*1.0981*	*0.8437*	*1.0292*	*0.8224*	*0.9801*	*0.3663*	*0.3213*	*0.3607*	*0.3298*	*0.3520*	*0.3292*
2	*0.8348*	*1.1549*	*0.8200*	*1.0456*	*0.8068*	*0.9856*	*0.3565*	*0.3157*	*0.3575*	*0.3277*	*0.3539*	*0.3309*
3	*0.8228*	*1.2259*	*0.8080*	*1.0737*	*0.8004*	*1.0007*	*0.3567*	*0.3223*	*0.3619*	*0.3350*	*0.3616*	*0.3399*
4	*0.8227*	*1.3050*	*0.8046*	*1.1093*	*0.8006*	*1.0222*	*0.3631*	*0.3355*	*0.3710*	*0.3477*	*0.3731*	*0.3534*
5	*0.8312*	*1.3883*	*0.8075*	*1.1492*	*0.8054*	*1.0474*	*0.3733*	*0.3521*	*0.3830*	*0.3635*	*0.3868*	*0.3693*
6	*0.8462*	*1.4734*	*0.8148*	*1.1913*	*0.8134*	*1.0748*	*0.3855*	*0.3698*	*0.3965*	*0.3805*	*0.4016*	*0.3864*
7	*0.8658*	*1.5589*	*0.8252*	*1.2345*	*0.8235*	*1.1033*	*0.3988*	*0.3877*	*0.4107*	*0.3978*	*0.4168*	*0.4037*
8	*0.8887*	*1.6440*	*0.8376*	*1.2778*	*0.8350*	*1.1320*	*0.4122*	*0.4049*	*0.4250*	*0.4147*	*0.4319*	*0.4208*
9	*0.9141*	*1.7283*	*0.8514*	*1.3207*	*0.8472*	*1.1605*	*0.4255*	*0.4213*	*0.4390*	*0.4310*	*0.4467*	*0.4373*
10	*0.9411*	*1.8115*	*0.8661*	*1.3631*	*0.8597*	*1.1886*	*0.4384*	*0.4366*	*0.4525*	*0.4464*	*0.4609*	*0.4530*

Figure No.4.1

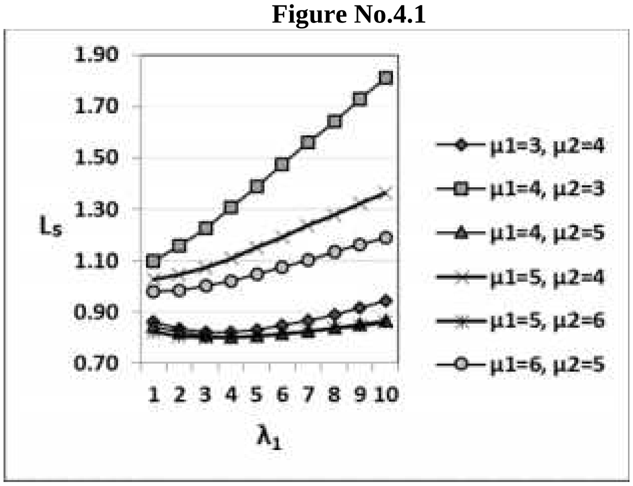
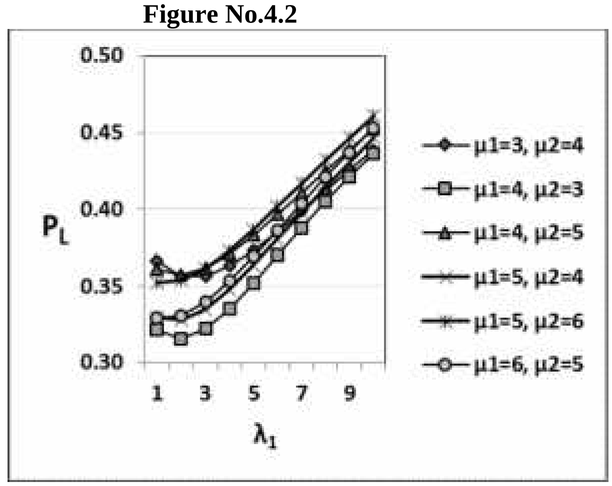


Figure No.4.2



8. Conclusion

The tables (1 – 4) reflect the expected queue length in the system and the loss probability of units. The corresponding curves are given in figures (1.1 – 4.2). On observing the numerical values and curves, it is remarked that the expected queue length in the system increases when the mean arrival rate increases. Likewise, for increasing mean arrival rates, loss probability of units increases.

In the case of fixed mean arrival rate and mean service rate in server 1, expected queue length decreases for increasing the mean service rate in sever 2. On the other hand, the expected queue length increases for increasing the mean service rate in server 1 when mean arrival rate and mean service rate in server 2 are fixed.

The expected queue length in the system under $«_1 > «_2$ is greater than that under $«_1 < «_2$ irrespective of mean arrival rates. The loss probability of units under $«_1 > «_2$ is less than that for the case $«_1 < «_2$. Suppose there are three or more servers provide services, the researcher is interested to obtain the minimization of loss probabilities. Based on the duration of service times, if one may consider as fast and slow service providers, then the researcher may get different results and minimum loss probability.

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