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AN EFFICIENT GRAPHICAL MODEL FOR ANALYZING AND DETERMINING THE STRUCTURAL OBJECTS WITH VARIOUS DIMENSIONS

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Abstract
Graph theory is mostly used to find more solutions in mathematical model. To make more enhancements in these fields the structural arrangements of various objects or technologies are needed for new inventions and modifications in the existing environment. In 1735, using the Koingsberg bridge problem the field graph theory gets started. This paper depicts some of the applications of graph theory in Dominating Set, Covering and Equitable Graph fields.

Key Words: Dominating Set, Covering and Equitable Graph.

Introduction

There are more categories of domination parameters have given by various authors. Acharya B.D., Sampathkumar E., and Waliker H.B. are the Indian Mathematicians who made a meaningful contribution to the enhancement of domination in graphs. The technical report given in 1979 titled "Recent developments in the theory of domination in graphs, Technical Report 14 MRI" that triggered a way to make more enhancements in Graph theory research.

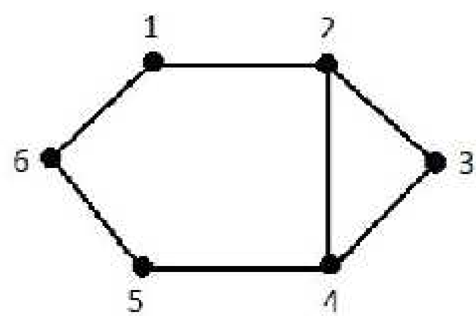
Swaminathan *et al*[10] considers real world problems that is given below and introduced the concept of equitable domination in graphs. Nodes with nearly equal capacity interact with each other in a consistent way in a network. Persons with equal status in a society have a friendly relationship. Employees with equal powers form association and have a close relationship in an Industry. In a democratic country, it's goal is to make citizens in terms of wealth, health, status must be equal and they have equal right is the goal. Thus, to study and implement this in a practical way, a graphical model is to be created.

This article deals with the concepts of Equitable Covering, Equitable Packing, Equitable Full Sets and Adjacency Inherent Equitable Graph have been depicted with basic examples and theorems.

Dominating Set

Let G = (V, E) be a graph. A subset D of V is called a dominating set of G if every vertex not in D (i.e.) (every vertex in V-D) is adjacent to some vertex in D**.**

Example



$D_1=\{2,3,5,6\}$ $D_2=\{1,3,4,6\}$ $D_3=\{1,2,4\}$ $D_4=\{2,4,5\}$ are dominating sets.

Minimal Dominating Set

A minimal dominating set in a graph G is a dominating set that contains no dominating set as a proper subset. (i.e) D is a minimal dominating set if D-$\{v\}$ is not a dominating set for any $v \in D$. In the above example, the vertex set D_1,D_2,D_3,D_4 are minimal dominating sets.

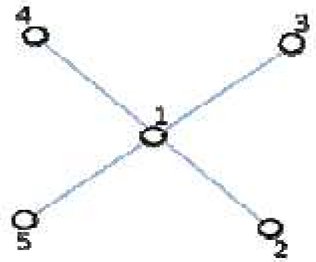
Minimum Dominating Set

A dominating set D is called a minimum dominating set, if D consists of minimum number of vertices among all dominating sets.

Domination Number γ(G)

The minimum cardinality of a minimal dominating set is called the domination number of G and is denoted by $\gamma(G)$

Example



$D_1=\{1\}$,$D_2=\{2,3,4,5\}$.

D_1 and D_2 are minimal dominating sets.

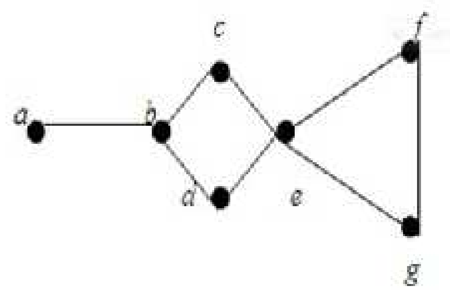
$D_1=\{1\}$ is the minimum dominating set since it has minimum number of vertices than D_2.

γ(G)=1 is the domination number of G.

Independent Set

A subset D of V is called an independent set of G if no two vertices of D are adjacent in G.

Example



$D_1=\{a,c,d\}$$D_2=\{a,c,d,f\}$$D_3=\{b,f\}$$D_4=\{b,g\}$ are an independent sets.

Maximal Independent Set

A maximal independent set is an independent set to which no other vertex can be added without destroying its independent property.

From the above example $D_2=\{a,c,d,f\}$is a maximal independent set.

Independence Number β(G)

The number of vertices in a maximum independent set of G is called the independence number of G and it is denoted by β(G).

Independent Dominating Set

A set D of vertices in a graph G is called an independent dominating set if D is both independent and dominating set of G.

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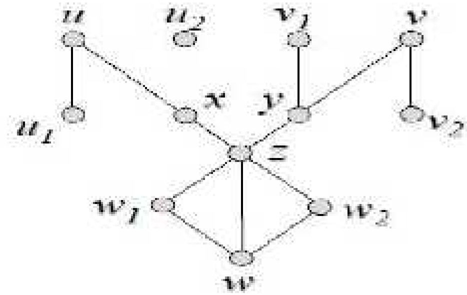
Example



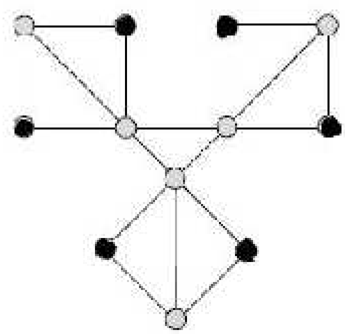
$S_1=\{u_1, u_2, v_1, v_2, w_1, w_2\}$ $S_2=\{x, y, z\}$ $S_3=\{u, v, w\}$.

S_1 and S_3 are independent dominating set. S_2 is dominating set but not independent dominating set.

Maximal Independent Dominating Set

An independent dominating set D is called a maximal independent dominating set, if every vertex not in D is adjacent to a vertex of D.

Example

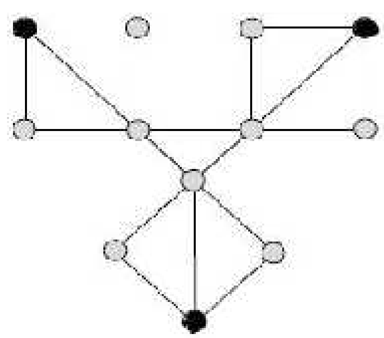


$S_1=\{u_1, u_2, v_1, v_2, w_1, w_2\}$ is a maximal independent dominating set.

Minimum Independent Dominating Set

An independent dominating set D is called a minimum independent Dominating set, if D consists of minimum number of vertices among all independent dominating sets.

Example

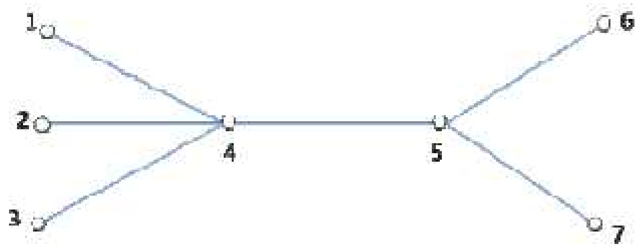


$S_3=\{u, v, w\}$ is a minimum independent dominating set.

Independent Domination Number i(G)

The independent domination number i(G) of G is the minimum cardinality among all independent dominating set of G.

Example



D_1={1,2,3,5},D_2={4,6,7} are independent dominating sets.

D_3={1,2,3,6,7} is a maximal independent dominating set.

D_2={4,6,7} is the minimum independent dominating set since it has minimum number of vertices than D_1.

i(G)=3 is the independent domination number of G.

Equitable Domination Set

A subset D of V is called an equitable dominating set if for every v $\in$ V$-$D there exists a vertex u $\in$D such that uv$\in$ E(G) and $|\deg(u) - \deg(v)| \le 1$.

Equitable Domination Number γ^e

The minimum cardinality of an equitable dominating set of G is called the equitable domination number of G and is denoted by γ^e.

Degree Equitable

A vertex u $\in$ V is said to be degree equitable with a vertex v $\in$ V $|\deg(u) - \deg(v)| \le 1$.

A Minimal Equitable Dominating Set

An equitable dominating set D is set to be a minimal equitable dominating set if no proper subset of D is an equitable dominating set.

Γ^e – set

A minimal equitable dominating set of maximum cardinality is called a Γ^e- set and its cardinality is denoted by Γ^e.

1-Minimal

An equitable dominating set is said to be 1 - minimal if *D*−*v* is not an equitable dominating set for all *v* $\in$*D*.

Equitable Isolates

If a vertex u$\in$V be such that $|\deg(u) - \deg(v)| \ge 2$ for all v$\in$N(u) then u is in every equitable dominating set. Such points are called equitable isolates.

Let I_e denote the set of all equitable isolates and I_s is the set of all isolated points of G and its$I_s \subseteq I_e \subseteq$ D for every equitable dominating set D.

Equitable Neighborhood

Let u$\in$V. The equitable neighborhood of u denoted by $N^e(u)$ is defined as $N^e(u) = \{u \in V / v \in N(u), |\deg(u) - deg(v) \le 1|\}$ and u$\in I_e \Leftrightarrow N^e(u) = \phi$.

The cardinality of $N^{e}(u)$ is denoted by $deg^{e}_{G}(u)$.

Maximum Equitable degree
The maximum equitable degree of a point in G is denoted by Δ^{e}(G).

$$\Rightarrow \Delta^{e}(G) = \max_{u \in V(G)} |N^{e}(u)|.$$

Minimum Equitable degree
The minimum equitable degree of a point in G is denoted by $\delta^{e}(G)$

$$\Rightarrow \delta^{e}(G) = \min_{u \in V(G)} |N^{e}(u)|.$$

Equitable Independent Set
A subset S of V is called an equitable independent set, if for any u$\in$S,

$$v \notin N^{e}(u) \text{ for all } v \in S - \{u\}.$$

Example
1. Any set S of cardinality 1 is an equitable independent set.
2. Every independent set is an equitable independent set.

Covering
A subset K of V such that every edge of G has at least one end vertex in K is called a covering of G.

Minimum Covering
A Covering K is minimum if G has no covering K' with $|K'|<|K|$.

Edge Covering
An edge covering of G is a subset L of E such that each vertex of G is an end Vertex of some edge in L.

A graph G has an edge covering if and only if $\delta(G) > 0$.

Covering Number $\alpha(G)$

The number of vertices in a minimum covering of G is called the covering number of G ant it is denoted by $\alpha(G)$.

Result
Any one vertex in a complete graph constitutes a minimal dominating set.

Result
Every dominating set contains at least one minimal dominating set.

Result
A graph may have many minimal dominating sets, and of different sizes.

Result
A minimal dominating set may or may not be independent.

Result
An independent set has the dominance property only if it is a maximal independent set. Thus an independent dominating set is the same as a maximal independent set.

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The Adjacency Inherent Equitable Graph of a Graph

Let $G = (V, E)$ be a simple graph. Let H be the graph constructed from G as follows $V(H) = V(G)$, two points u and v are adjacent in H if and only if u and v are adjacent and degree equitable in G. H is called the Adjacency Inherent Equitable Graph of G or equitable associate of G and is denoted by $e(G)$.

Result

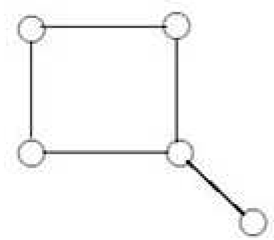
1. $e = uv \in E(e(G))$.Then *u* and *v* are adjacent and degree equitable in *G*. Therefore $e \in E(G) \cdot E(e(G)) \subseteq E(G)$
2. An edge $e = uv \in E(G)$ is said to be equitable if $|d(v) - d(u)| \leq 1$.
3.

Let $E^e(G)$ be the set of all equitable edges of *G*. Then clearly, $E^e(G) = E(e(G))$.

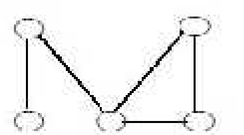
Result

$\overline{e(G)}$ need not be equal to $e(\bar{G})$.

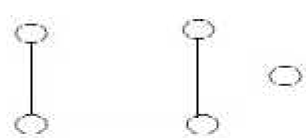
For consider G



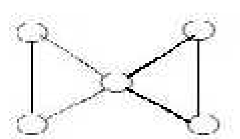
$\overline{G}$:



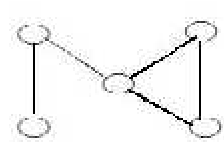
$e(G)$:



$e(G)$:



$e(\overline{G})$:



Therefore $\overline{e(G)} \neq e(\bar{G})$.

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Result

$$\text{Since } E(e(G)) \subseteq E(G) \text{ we have } E(\overline{e(G)}) \supseteq E(\bar{G}) \supseteq E(e(\bar{G})).$$

Example

Let $G_1 = K_3, G_2 = K_8, G_3 = K_4$. Join every point of G_1 with every point of G_2 as well as G_3. Let G be the resulting graph. Then $|V(G)| = 15, \gamma^e(G) = 3, \gamma^e(\bar{G}) = 15$,

$$\gamma^e(G) + \gamma^e(\bar{G}) = 18 > |V(G)| + 1, \gamma^e(G) \cdot \gamma^e(\bar{G}) = 45 > |V(G)|.$$

Theorem

For any graph G such that G and $\bar{G}$ have no equitable isolates,

$$\gamma^e(G) + \gamma^e(\bar{G}) \leq n.$$

Proof

Since G and $\bar{G}$ have no equitable isolates, $\gamma^e(G) \leq \left\lfloor \frac{n}{2} \right\rfloor, \gamma^e(\bar{G}) \leq \left\lfloor \frac{n}{2} \right\rfloor$.

Therefore $\gamma^e(G) + \gamma^e(\bar{G}) \leq \left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor \leq n$.

Theorem

Let G be a graph in which every edge in equitable. Then

1. $\gamma^e(G) \cdot \gamma^e(\bar{G}) \leq n$.
2. If in addition G and $\bar{G}$ have no isolates, then $\gamma^e(G) + \gamma^e(\bar{G}) \leq \frac{n}{2} + 2$.

Proof

1. Suppose G is a graph in which every edge is equitable. Then $\gamma^e(G) = \gamma(e(G))$. Also, $\overline{e(G)} = e(\bar{G})$. Since every edge of G is equitable, every edge of $\bar{G}$ is equitable and hence $\gamma^e(\bar{G}) = \gamma(e(\bar{G}))$. $\gamma(e(G)) \cdot \gamma(\overline{e(G)}) \leq n$.

$$\Rightarrow \gamma^e(G) \cdot \gamma(e(\bar{G})) \leq n. \text{(Since } e(\bar{G}) = (\overline{e(G)})\text{)}.$$

$$\Rightarrow \gamma^e(G) \cdot \gamma^e(\bar{G}) \leq n. \text{(Since } \gamma^e(\bar{G}) = \gamma(e(\bar{G}))\text{)}.$$

2. Suppose G and $\bar{G}$ have no isolates. Then $e(G)$ and $\overline{e(G)}$ have no isolates. Therefore $e(G)$ and $\overline{e(G)}$ have no isolates. $\gamma(e(G)) + \gamma(\overline{e(G)}) \leq \frac{n}{2} + 2$.

$$\Rightarrow \gamma(e(G)) + \gamma(e(\bar{G})) \leq \frac{n}{2} + 2.$$

$$i.e \quad \gamma^e(G) + \gamma^e(\bar{G}) \leq \frac{n}{2} + 2.$$

Theorem

Let G be a graph with $\delta^e(G) \geq 2$ and let G be not in B. where B is the set of graphs given in figure 3.1-B.

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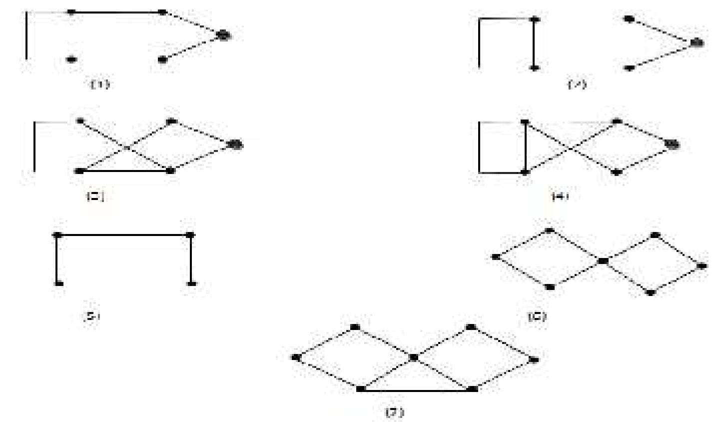

FIGURE 3.1 - A

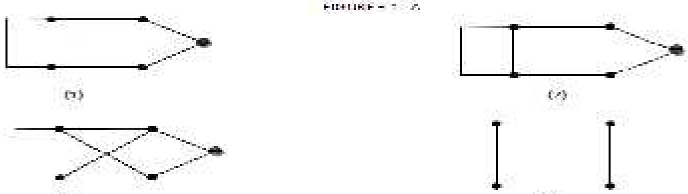

FIGURE 3.1 - B

Then $\gamma^e(G) \leq \frac{2n}{5}$.

Proof

Let A be the set of graphs given in figure 3.1-A . If G does not belong to B, then $e(G)$ does not belong to A. Therefore $\gamma\left(e(G)\right) \leq \frac{2n}{5}$.But $\gamma^e(G) = \gamma\left(e(G)\right)$.

Therefore $\gamma^e(G) \leq \frac{2n}{5}$.

Theorem

Let A and B be the set of graphs specified in Fig 3.1-A and Fig 3.1-B.Let G be a connected graph with $\delta^e(G) \geq 2$and $\gamma^e(G) = \left\lfloor \frac{n}{2} \right\rfloor$.Then $G \in B \cup C$, where C is the set of graphs in figure 3.1-C.

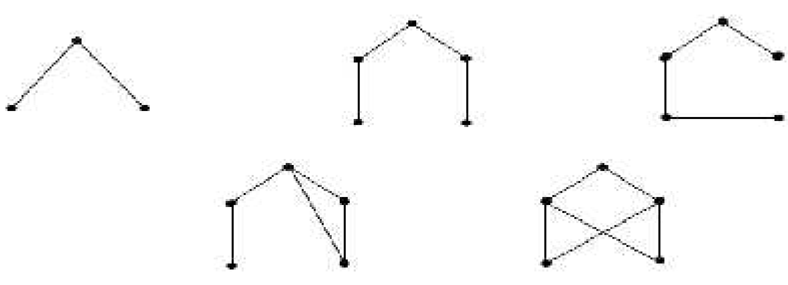

FIGURE 3.1 - C

Proof

Let G satisfy the conditions of the theorem. Then$\delta\left(e(G)\right) \geq 2$ and

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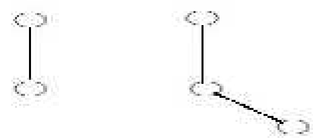
$\gamma(e(G)) = \left\lfloor \frac{n}{2} \right\rfloor$.Therefore $e(G) \sqsubseteq A \cup C$. $e(G) \sqsubseteq A$ if and only if G$\sqsubseteq$B and $e(G) \sqsubseteq C$ if and only if G$\sqsubseteq C$. Hence the theorem is proved.

Result

If G is connected graph, then $e(G)$ need not be connected.

Example



Figure

Then $e(G)$ has an isolated vertex and hence not connected. If $e(G)$ is connected then G is connected.

Theorem

If e(G) is connected and $\delta^e(G) \geq 3$, then $\gamma^e(G) \leq \frac{3n}{8}$.

Proof

Since $e(G)$ is connected and $\delta^e(G) \geq 3, \delta(e(G)) \geq 3$.

Therefore $\gamma(e(G)) \leq \frac{3n}{8}$.

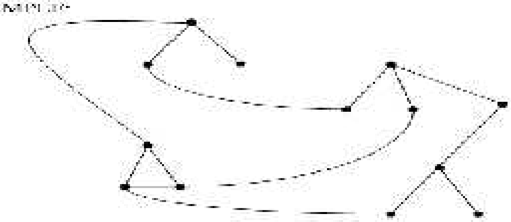
$$\Rightarrow \gamma^e(G) \leq \frac{3n}{8}.$$

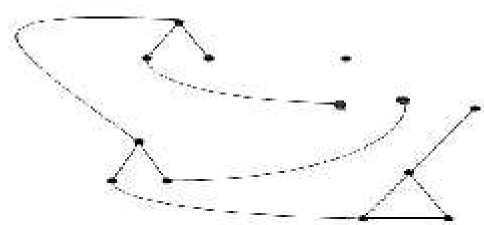
Result

1. If G is claw-free then $e(G)$ is claw-free.

For suppose G is claw-free. Suppose $e(G)$ is contains a claw as an induced sub graph. Then G is not claw-free since $E(e(G)) \sqsubseteq E(G)$.Therefore $e(G)$ is claw-free.

2. There exit graphs G such that G and $e(G)$ are connected $e(G)$ is claw free and G contains a claw.





3. If G contains a claw and if the edges of one claw are not equitable then $e(G)$ will not contain that claw.

A claw(net) in G is called a degree equitable claw(net) or simply equitable claw(net) if and only if all the edges of the claw(net) are degree equitable.

Theorem

Let G and $e(G)$ be connected. If G is equitable claw-free and equitable net-free then $\gamma^e(G) \leq \left|\frac{n}{3}\right|$.

Proof

It is obvious that, If G is a connected claw-free and net-free graph then $\gamma^e(G) \leq \left|\frac{n}{3}\right|$.

Conclusion

There is a scope for making a detailed study on the characteristics of graphs like equitable covering, Equitable full sets, Adjacency Inherent Equitable graphs. Using this Graphical model structural analysis for all the common problems that is to be applicable for basic graphical theory models and for measuring the cardinality of various dominating set.

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