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AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPONENTIAL DISTRIBUTION AND SELLING PRICE DEPENDENT DEMAND UNDER PERMISSIBLE DELAY IN PAYMENTS

A. Lakshmana Rao[1], K.P.S. Suryanarayana[2], B. Neela Rao[3], B. Venkata Rao[4]
[1-4]Department of Basic Sciences and Humanities, Aditya Institute of Technology and Management, Tekkali, India.

Abstract
In this paper, we have developed an inventory model is constructed for deteriorating items with instantaneous replenishment, exponential decay rate and selling price dependent demand without shortages under permissible delay in payments. It is assumed that a constant fraction of the on-hand inventory deteriorates per unit of time. The exact formulae of the optimal average cost and the lot size are derived without carrying out any approximation over the deteriorating rate. Different decision making situations illustrated with the help of numerical examples.

Key Words: Selling Price Dependent, Exponential Distribution, Economic Order Quantity, Permissible Delay In Payments.

INTRODUCTION

In classical economic order quantity (EOQ) inventory models were developed under demand is the major factor in the inventory management. Therefore, decisions of inventory are to be made because of the present and future demands. As demand plays a key role in modeling of deteriorating inventory, researchers have recognized and studied the variation (or their combinations) of demand from the view point of real life situations. Demand may be constant, time-varying, stock-dependent and price-dependent, etc. The constant demand is valid only when the phase of the product life cycle is matured and also for finite periods of time. Covert and Philip (1973), Misra (1975), Dave (1979) and Sarma (1987), etc., established inventory models with constant demand rate. EOQ models for deteriorating items with trended demand were considered by Goswami and Chaudhuri (1991, 1992), Xu and Wand (1990), Chund and Ting (1993, 1994), Jalan and Chaudhuri (1999) and Lin et al. (2000), etc. In this extent, Begum et al, (2009) developed an inventory model with exponential demand rate, finite production rate and shortages. Silver and Meal (1969) developed an approximate solution procedure for the general case of time-varying demand. Generally, this type of demand exists for some particular goods. Many research articles by Dave and Patel (1981), Sachan (1984), Deb and Chaudhuri (1986) and Hargia (1993), etc., analyzed linear time varying demand. Later, Ghosh and Chaudhuri (2004, 2006), Kharna and Chaudhuri (2003) and Begum et al. (2010), etc., established their models with quadratic time-varying demand. Tend and Chang (2005) established an economic production quantity models for deteriorating items with demand depend on price and stock. The deterioration rate of inventory in stock during the storage period constitutes an important factor which has attracted the attention of researchers. In inventory problems, deterioration is defined as damage, decay, spoilage, evaporation, obsolescence and loss of utility or loss of marginal value of goods that results in decrease the usefulness of the original one. Whitin (1957) is the first researcher who studied an inventory model for fashion goods deteriorating at the end of a prescribed storage period.

The assumption of the constant deterioration rate was relaxed by Covert and Philip (1973) who used a two parameter Weibull distribution to represent the distribution of time to deterioration. This model was further generalized by Philip (1974) by taking three parameter Weibull distribution deterioration. Shah and Jaiswal (1977) established orderlevelinventory modelfor perishable items with a constant rate of deterioration. Recently, Begum et al. (2010) has discussed an EOQ model for the deteriorating items with two parameter Weibull distribution deterioration. The problem of determining the Economic Order Quantity (EOQ) under the condition of a permissible delay in payment has drawn the attention of researchers in recent times. It is assumed that the supplier (wholesaler) allows a delay of a fixed period for settling the amount owed to him/her. There is no interest charged on the outstanding amount if it is paid within the permissible delay period.

Beyond this period, interest is charged. During this fixed period of permissible delay in payments, the customer (a retailer) can sell the items, invest the revenues in an interest – earning account and earn interest instead of paying off the over – draft which is necessary if the supplied requires settlement to the account immediately after replenishment. The customer finds it economically beneficial to delay the settlement to the least moment of the permissible period of delay. This problem was first studied by Goyal (1985) for non-deteriorating items having a constant demand rate. Chand and Ward (1987) commented in a brief note on some of the assumptions made by Goyal (1985) in analyzing the cost of funds tied up in inventory. The effects of deterioration of goods in stock on the cost and price components cannot be ignored in practice. The model of Goyal (1985) and Aggarwal and Jaggi (1995) was extended by Gour Chandra Mahata (2011) to the case of deteriorating item. Hwang and Shinn (1997) discussed lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments when the demand rate would depend on retail price.

In the present study, the researchers assume that the time dependence of demand is linear. Deterioration rate is assumed to be exponential distribution. The rate of replenishment is infinite and shortages are not allowed. The results presented in this study extend and improve the corresponding results of Gour Chandra Mahata (2011) by taking into account a selling price dependent demand rate. The solution procedure involving different decision making situations is illustrated with the help of numerical examples.

Notations and Assumptions

Notations

a) The demand rate is dependent on selling price ‘s’ per unit, which is linear function i.e
$\lambda(s) = a - b\,s$ where a, b>0
b) θ = constant rate of deterioration.
c) A = Ordering cost per ordering.
d) c = unit purchasing cost per item.
e) I_e = Interest earned per rupees per year.
f) I_c = Interest changed per rupee in stocks per year.
g) M = permissible period (in years) of delay in settling the accounts with the supplier.
h) T = time interval (in years) between two successive only.

Assumptions

- The demand rate for the item is represented by a linear and continuous function.
- Replenishment rate is infinite and replenishment is instantaneous.
- The lead time is zero.
- Shortages are not allowed.
- The distribution of time to deterioration of an item follows the exponential distribution $f(t) = \theta e^{-\theta t}$, t>0

Where, θ is called the deterioration rate; a constant fraction θ, assumed to be small of the on-hand inventory gets deteriorated per unit item during the cycle time.

- There is no repair or replenishment of deteriorated units in the given cycle.
- When $T \geq M$, the account is settled at time T = M and retailer starts paying for the interest charges on the items in stock with rate I_c. When $T \leq M$, the account is settled at T = M and the retailer does not need to pay interest charge.
- The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

Mathematical Formulation

$$\frac{d}{dt}I(t) + \theta I(t) = -\lambda(s), 0 \leq t \leq T \quad (1)$$

Where I(0) = Q, I(T) = 0, λ(s) = a – b s and $f(t) = \theta e^{-\theta t}$

The solution of equation is

$$I(t) = \left(\frac{a-bs}{\theta}\right)\left(1 - e^{\theta(T-t)}\right) \quad (2)$$

The initial order quantity at t = 0 is

$$Q = I(0) = \left(\frac{a-bs}{\theta}\right)\left(1 - e^{\theta T}\right) \quad (3)$$

The total demand during one cycle is

$$\int_0^T (a-bs)dt = (a-bs)T \quad (4)$$

Number of deteriorated units is

= Q – (a – b s) T

$$= (a-bs)\left(\frac{1-e^{\theta T}}{\theta} - T\right) \quad (5)$$

The cost of stock holding for one cycle is

$= h\int_0^T I(t)dt,$ where h = ph_p

$$= \frac{h(a-bs)}{\theta}\left(T + \frac{1-e^{\theta T}}{\theta}\right)$$

Hence, the holding cost per unit time is

$$\frac{h(a-bs)}{\theta T}\left(T + \frac{1-e^{\theta T}}{\theta}\right) \quad (6)$$

Case (i) Let T>M

Since, the interest is payable during time (T-M), the interest payable in one cycle is:

$$= pI_p\int_M^T I(t)dt = \frac{pI_p(a-bs)}{\theta}\left((T-M) + \frac{1-e^{\theta(T-M)}}{\theta}\right)$$

Hence, interest payable per unit time is

$$\frac{pI_p(a-bs)}{\theta T}\left((T-M) + \frac{1-e^{\theta(T-M)}}{\theta}\right) \quad (7)$$

Interest earned per unit time is

$$= \frac{pI_e}{T}\int_0^T t\lambda(s)dt = \frac{pI_e}{2}(a-bs)T \quad (8)$$

Total variable cist per cycle is = Ordering cost + Cost of deteriorated units+ Inventory holding cost + interest payable beyond permissible period – interest earned during the cycle

Hence, total variable cost per unit time in this case is given by:

$$C_1(t) = \frac{A}{T} + \frac{p(a-bs)}{T}\left[\frac{1-e^{\theta T}}{\theta} - T\right] + \frac{h(a-bs)}{\theta T}\left[T + \frac{1-e^{\theta T}}{\theta}\right]$$

$$+ \frac{pI_p(a-bs)}{\theta T}\left[(T-M) + \frac{1-e^{\theta(T-M)}}{\theta}\right] - \frac{pI_e}{2}(a-bs)T \quad (9)$$

The researchers have now to minimize C_1(T) for a given value of M. The necessary and sufficient conditions to minimize C_1(T) for a given value of M are respectively.

$\frac{\partial C_1(T)}{\partial T} = 0$ and $\frac{\partial^2 C_1(T)}{\partial T^2} > 0$

$\frac{\partial C_1(T)}{\partial T}$ = 0 implies that

$$= -\frac{A}{T^2} + \left[\frac{(1-e^{\theta T})}{T^2} + \frac{e^{\theta T}}{T}\right]\left[p(a-bs) + \frac{h(a-bs)}{\theta^2}\right]$$
$$+ \frac{pI_p(a-bs)}{\theta}\left[\frac{M}{T^2} - \frac{1}{\theta}\left(\frac{1}{T^2} + e^{\theta(T-M)}\left(\frac{\theta}{T} + \frac{1}{T^2}\right)\right)\right] - \frac{pI_e}{2}(a-bs) = 0 \qquad (10)$$

By solving equation (10) for T, we obtain the optimal cycle length $T = T^*_1$ provided it satisfies the condition $\frac{\partial^2 C_1(T)}{\partial T^2} > 0$. The EOQ Q^*_0 for this case is given by:

$$Q^* = \left(\frac{a-bs}{\theta}\right)\left(1 - e^{\theta T^*_1}\right)$$

The minimum annual variable cost $C_1(T^*_1)$ is then obtain from Equation (9) for $T = T^*_1$

Case (ii) Let T<M

In this case, the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period for the items kept in stock. Interest earned up to T is

$$pI_e \int_0^T (a+bs)t\,dt = pI_e(a+bs)\frac{T^2}{2}$$

and interest earned during (M-T) i.e., up to permissible delay period is

$$pI_e(M-T)\int_0^T (a+bs)\,dt = pI_e(M-T)T(a+bs)$$

Hence, the total interest earned during the cycle is

$$pI_e(a+bs)\frac{T^2}{2} + pI_e(M-T)T(a+bs) = pI_e(a+bs)\left(\frac{T^2}{2} + (M-T)T\right) \qquad (11)$$

Total variable cost per cycle = Ordering cost + cost of deteriorating units + Inventory holding cost – Interest earned during the cycle.

Hence, the total variable cost per unit time is

$$C_2(T) = \frac{A}{T} + \frac{p(a-bs)}{T}\left[\frac{1-e^{\theta T}}{\theta} - T\right] + \frac{h(a-bs)}{\theta T}\left[T + \frac{1-e^{\theta T}}{\theta}\right]$$
$$-pI_e(a+bs)\left(\frac{T^2}{2} + (M-T)T\right) \qquad (12)$$

The researchers have now to minimize $C_2(T)$ for a given value of M. The necessary and sufficient conditions to minimize $C_2(T)$ for a given value of M are respectively.

$\frac{\partial C_2(T)}{\partial T} = 0$ and $\frac{\partial^2 C_2(T)}{\partial T^2} > 0$

$\frac{\partial C_2(T)}{\partial T} = 0$ this implies that

$$-\frac{A}{T^2} + \left[\frac{(1-e^{\theta T})}{T^2} + \frac{e^{\theta T}}{T}\right]\left[p(a-bs) + \frac{h(a-bs)}{\theta^2}\right] - pI_e(a-bs)(M-T) = 0 \qquad (13)$$

By solving equation (13) for T, we obtain the optimal cycle length $T = T^*_2$ provided it satisfies the condition $\frac{\partial^2 C_2(T)}{\partial T^2} > 0$. The EOQ Q^*_0 for this case is given by:

$$Q^* = \left(\frac{a-bs}{\theta}\right)\left(1 - e^{\theta T^*_2}\right)$$

The minimum annual variable cost $C_2(T^*_2)$ is then obtain from Equation (12) for $T = T^*_2$.

Case (iii) Let T=M

For T = M, both the cost functions $C_1(T)$ and $C_2(T)$ become identical and it is denoted by C(M), say and its is obtained on substituting T = M either in equation (9) or in (12). Thus

$$C(M) = \frac{A}{M} + \frac{p(a-bs)}{M}\left[\frac{1-e^{\theta M}}{\theta} - M\right] + \frac{h(a-bs)}{\theta M}\left[M + \frac{1-e^{\theta M}}{\theta}\right]$$
$$-pI_e(a+bs)\left(\frac{M^2}{2}\right) \qquad (14)$$

The EOQ is

$$Q^* = \left(\frac{a-bs}{\theta}\right)\left(1-e^{\theta M}\right)$$

Now in order to obtain the economic operating policy, the steps are to be followed:

- Step1: Determine T^*_1 from equation (10). If $T^*_1 \geq M$, obtain $C_1(T^*_1)$ from equation (9).
- Step2: Determine T^*_2 from equation (13). If $T^*_2 < M$, obtain $C_2(T^*_2)$ from equation (12).
- Step3: If $T^*_1 < M$ and $T^*_2 \geq M$ then, evaluate C(M) from equation (14).
- Step4: Compare $C_1(T^*_1)$, $C_2(T^*_2)$ and C(M) and take the minimum.

Numerical Examples

The numerical examples given, covers all the three cases that arise in this model:

Example 1 (Cases 1 and 2)

Let A = Rs.200, a = 50 units per year, b= 5 units per year, I_p = 0.15 per year, I_e = 0.13 per year, h = Re. 0.2, p = Rs 20, M = 3 year, θ = 0.2.

Solving equation (9), we have T_1^* = 5.027 and the minimum average cost is $C_1(T^*_1)$ = 258.047. Solving (12), the researchers have T_2^* = 5.01 and the minimum average cost is $C_2(T^*_2)$ = 243.25. Here T_1^* >M, T_2^* <M both hold and this implies that both the cases 1 and 2 hold. Now $C_2(T^*_2) < C_1(T^*_1)$. Hence, the minimum average cost in this case is $C_2(T^*_2)$ = 243.25 where the optimal cycle length is T_2^* = 5.01 year<M. The economic order quantity is given by Q^* = 88.921 units.

Example 2 (Case1)

Let A = Rs.200, a = 50 units per year, b= 5 units per year, I_p = 0.15 per year, I_e = 0.13 per year, h = Re. 0.2, p = Rs 20, M = 3 year, θ = 0.01.

Solving equation (9), we have T_1^* = 5.066 and the minimum average cost is $C_1(T^*_1)$ = 321.696. Solving (12), the researchers have T_2^* = 4.67 and the minimum average cost is $C_2(T^*_2)$ = 1956.

Here T_2^* >M which contradicts case 2. In this case T_2^*>M which is case 1. Therefore, the minimum average cost in this case is $C_1(T^*_1)$ = 321.696, the EOQ is Q^* = 1424.45.

Example 3 (Case 2)

Let A = Rs.200, a = 50 units per year, b= 5 units per year, I_p = 0.15 per year, I_e = 0.13 per year, h = Re. 0.2, p = Rs 10, M = 3 year, θ = 0.2.

Solving equation (9), we have T_1^* = 5.012 and the minimum average cost is $C_1(T^*_1)$ = 150.643. Here T_1^* <M which contradicts case 1. Again solving (12), the researchers have T_2^* = 4.17 and the minimum average cost is $C_2(T^*_2)$ = 143.38. In this case T^*<M which is case 2. Therefore, the minimum average cost in this case is $C_2(T^*_2)$ = 143.38, the EOQ is Q^* = 88.710.

Example 4 (Case3)

Let A = Rs.200, a = 100 units per year, b= 5 units per year, I_p = 0.5 per year, I_e = 0.01 per year, h = Re. 0.12, p = Rs 10, M = 2.32 year, θ = 0.3.

In this case $T_1^* = T_2^* = 2.32 = M$ which is case 3. The optimal cost in this case is C(M) = 3256.56 and the EOQ is $Q^* = 119.28$ units for M = 2.32 year.

CONCLUSION

The present model seeks on extension of Gour Chandra Mahata (2011) work by taking a selling price dependent demand rate into consideration. This consideration makes the model more realistic. A selling price dependent rate implies a steady increase in the demand of the product. This model can be used for items like fruits and vegetables whose deterioration rate increases with price. This type of demand pattern is observed in the market in the case of many products. Several numerical illustrations are given to explain the solution procedure of the model. The suggested model can be extended for items having stock dependent demand, power demand.

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