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A STUDY ON MATRIX PRODUCT (MOD-n) OF ADJACENCY MATRICES

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Abstract
In this paper, I found a new adjacency matrix using Fusion Algorithm for the connected graph. Also I present the realization of the product of adjacency matrices under ***mod-n*** *is graphical, if n≤2 and which is proved by an example of the product of adjacency matrices under* ***mod-3*** *is not graphical.*

Key Words: Adjacency Matrices, Connectedness, Mod-N.

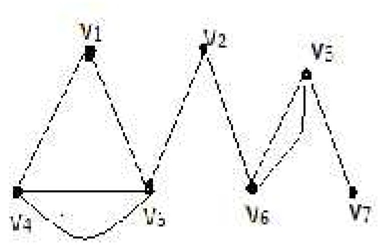
Introduction

Graphs considered in this paper are undirected. Let G be a graph on the set of vertices {v_1, v2... vn}.Two vertices v_i and v_j, i≠j, are said to be adjacent to each other if there is an edge between them. An adjacency between the vertices v_i and v_j is denoted by $v_i \sim_G v_j$, and $v_i \nsim_G v_j$ denotes that v_i is not adjacent with v_j in the graph G. The adjacency matrix of G is a matrix A (G) = (a_{ij}) ∈ M_n (ℝ) in which a_{ij} = 1 if v_i and v_j are adjacent, and a_{ij} = 0 otherwise. In Manjunatha Prasad [9&10], the ordinary matrix product was considered and some properties of graphs G and H were studied for the realization of A(G) A(H). A necessary and sufficient condition for such a realization was that H must be a sub graph of complement of G and for each ordered pair of distinct vertices.

Algorithm: To Find New Adjacency Matrix after Fusion
Step1: Change u's row to the sum of u's row with v's row and (symmetrically) change u's column to the sum of u's column with v's column.
Step2: Delete the row and column corresponding to v. The resulting matrix is the adjacency matrix of the new graph G.

Algorithm: Fusion Algorithm for Connectedness
Step1: Replace G by its underlying simple graph. To get adjacency matrix of new graph just replace all non zero entries off the diagonal by 1 and make all entries on the diagonal 0.Denote the underlying simple graph also as G.
Step2:Fuse vertex v_1 to the first of the vertices v_1.......v_n with which it is adjacent to give a new graph, also denoted by G, in which the new vertex is also denoted by v_i.
Step3: The above two step process gives the adjacency matrix X (G).
Step4: Repeat steps 1 and 2 with v_1 will v_2 is not adjacent to any of the other vertices.
Step5: Repeat step 2 and 4 on the vertex v_2 of the last graph and then on all remaining vertices of the resulting graphs. The final graph is empty and the number of its (isolated) vertices is the number of connected components of the initial graph G.

Example
Given below is the ***adjacency matrix*** of graph G with seven vertices listed as v_1, v2, v3, v4, v5, v6, v7. Use fusion algorithm to check the connectedness.



G

$$A(G)=\begin{bmatrix}0&0&0&1&1&0&0\\0&0&0&0&1&1&0\\0&0&0&0&0&2&1\\1&0&0&0&2&0&0\\1&1&0&2&0&0&0\\0&1&2&0&0&0&0\\0&0&1&0&0&0&0\end{bmatrix}$$

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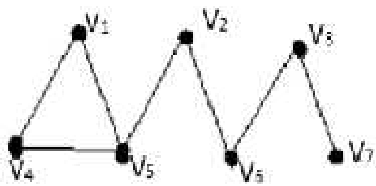
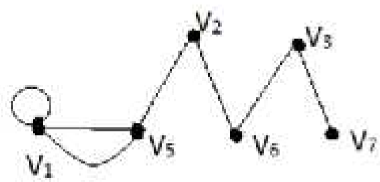
Step1: Replacing G by Its Underlying Simple Graph



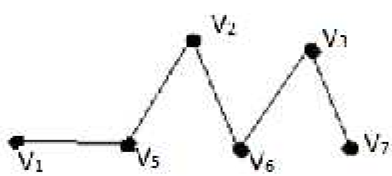
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fusing v_4 with v_1



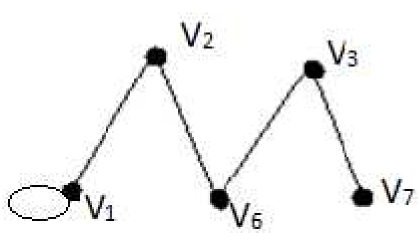
$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Repeating Step 1



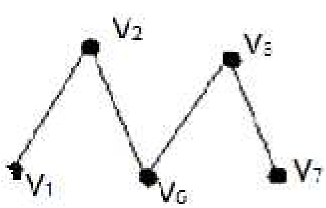
$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Fusing v_5 with v_1



$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Repeating Step 1



$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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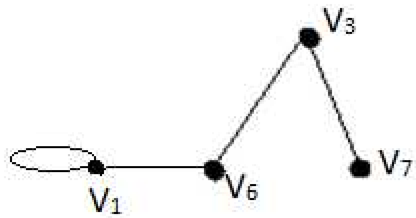
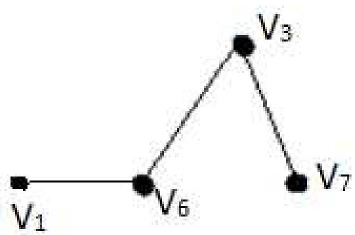
Fusing v_2 with v_1



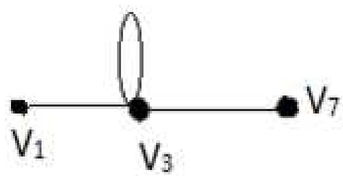
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Repeating Step 1



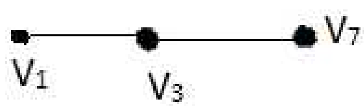
$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Fusing v_6 with v_3



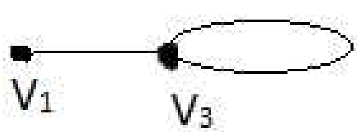
$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Repeating Step 1



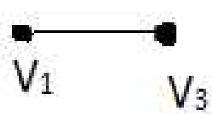
$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Fusing v_7 with v_3



$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Repeating Step 1



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Fusing v_3 with v_1

$$[1]$$

Repeating Step 1

$$[0]$$

Since the final adjacency matrix is 1×1 null matrix.
We conclude that the original graph G has ***1 connected component***.

Now we given two graphs G and H on the same set of vertices {v_1, v2… vn}. The matrix product is considered over **modulo-2.** In fact, the class of graphs H, for which A (G) A (H) (**mod-2**) is graphical, is found to be a larger class. In the following Example 1, we have a graph H such that A (G) A (H) (**mod-2**) is graphical but H is not a sub graph of $\overline{G}$.

Example 1 (mod 2 i.e; only 0 and 1)

In the following graphs G and H (as shown in Figure 2) on seven vertices, H is not a sub graph of $\overline{G}$. Note that the adjacency matrices of G and H are

$$A(G) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \text{ and } A(H) = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{Respectively, and } A(G)\,A(H) = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Clearly, A (H) A (H) is graphical and K_7 is the graph realizing A (G) A (H).

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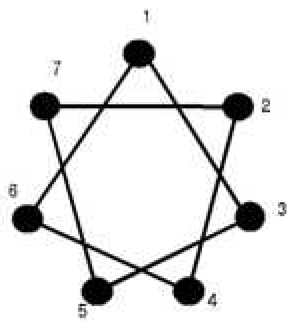

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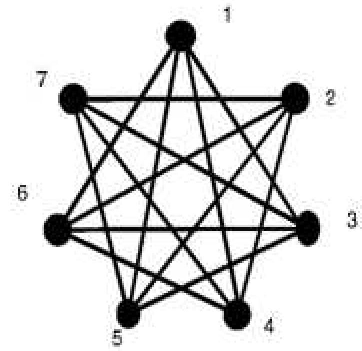
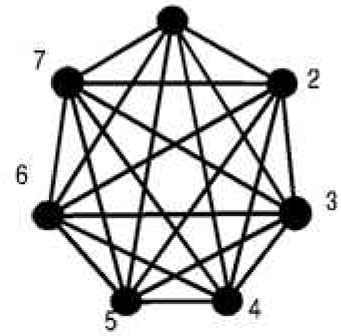


H



K_7

Theorem 1.1

The product A (G) A (H) is graphical if and only if the following statements are true according to K. Manjunath [9.]

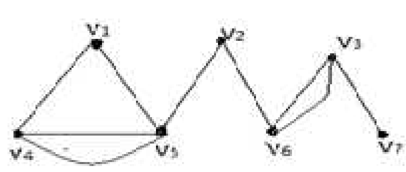
1. The diagonal entries of the matrix product A (G) A (H) are zeros if and only if for each vertex v_i, the cardinality of the set of vertices $\{ v_k : v_k \sim_G v_i , v_k \sim_H v_i \}$ is even.
2. The (i , j)th ($i \neq j$) entry of the matrix product A(G) A(H) is either 0 or 1 depending on whether the number of GH paths from v_i to v_j is even or odd, respectively.
3. The matrix product A(G) A(H) is symmetric if and only if for each pair of distinct vertices v_i and v_j, the numbers of GH paths and HG paths from v_i to v_j have the same parity (both are even or both are odd).
4. For every i ($1 \leq i \leq n$), there are even number of vertices v_k such that $v_i \sim_G v_k$ and $v_k \sim_H v_i$.
5. For each pair of vertices v_i and v_j ($i \neq j$), the numbers of GH paths and HG paths from v_i to v_j have same parity.

Given two graphs G and H on the same set of vertices $\{v_1,v_2,\ldots,v_n\}$.When the matrix product is considered over **modulo-3**, this condition would not remain as a necessary condition.

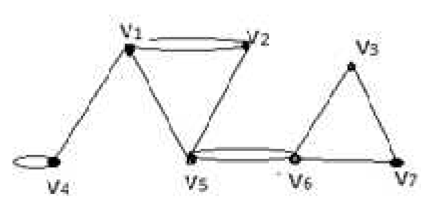
Example 2 (mod 3 i.e.; **0, 1** and **2)**

In the following graphs G and H (as shown in Figure 2) on seven vertices, H is not a sub graph of $\overline{G}$. Note that the adjacency matrices of G and H are

$$A(G) = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and } A(H) = \begin{bmatrix} 0 & 2 & 0 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$



G



H

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Respectively, and $A(G)A(H) = \begin{bmatrix} 2 & 1 & 0 & 2 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 2 \\ 2 & 2 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

By the theorem 1.1 A (G) A (H) is not graphical.
Hence the product of adjacency matrices under **mod-n** is graphical, if $n \leq 2$.

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