SOLVING MULTI-OBJECTIVE LINEAR TRANSPORTATION PROBLEM USING LINEAR SOLUTION TECHNIQUES

Dr. M. S. Annie Christi*   R. Sumitha Devi**
**Associate Professor, Department of Mathematics,Providence College for Women, Coonoor, India.*
***Department of Mathematics, Providence College for Women, Coonoor, India.*

Abstract
This paper is focused on the solution of the Multi-Objective Linear Transportation Problem using compensatory fuzzy approach via Fuzzy programming Algorithm. Here Supply and Demand are Trapezoidal Fuzzy Numbers. By using linear solution techniques, our approach generates compromise solutions which are both compensatory and Pareto-optimal. We use the hyperbolic membership function .The results of the problem reveal that if we use the hyperbolic membership function, then the crisp model becomes linear. This is illustrated by an example.

Keywords:Compensatory fuzzy approach, Multi-Objective transportation problem, Fuzzy programming algorithm, hyperbolic membership function.

1. Introduction

The transportation problem is a special type of linear programming problem and it has wide practical applications in manpower planning, personnel allocation, inventory control, production planning, etc. The parameters of the transportation problem are unit costs (or profits), supply and demand quantities. The unit cost c_{ij} is the coefficient of the objective function and it could represent transportation cost, delivery time, number of goods transported unfulfilled demand, and many others. Thus multiple objectives may exist concurrently which lead to the research work on multi-objective transportation problems (MOTP). Also in practice, the parameters of MOTP are not always exactly known and stable. This imprecision may follow from the lack of exact information, changeable economic conditions, etc. A frequently used way of expressing the imprecision is to use the fuzzy numbers.

Hussein **[9]** deal with the complete solutions of MOTP with possibility coefficients. Das *et al*. **[6]** focused on the solution procedure of the MOTP where all the parameters have been expressed as interval values by the decision maker. Ahlatcioglu *et al*. **[2]** proposed a model for solving the transportation problem that supply and demand quantities are given as triangular fuzzy numbers bounded from below and above, respectively. Basing on extension principle, Liu and Kao **[14]** developed a procedure to derive the fuzzy objective value of the fuzzy transportation problem where the cost coefficients, supply and demand quantities are fuzzy numbers. Using signed distance ranking, defuzzification by signed distance, interval-valued fuzzy sets and statistical data, Chiang **[5]** get the transportation problem in the fuzzy sense. Ammar and Youness **[3]** examined the solution of MOTP which has fuzzy cost, source and destination parameters. They introduced the concepts of fuzzy efficient and parametric efficient solutions. Islam and Roy **[10]** dealt with an multi-objective entropy transportation problem with an additional delivery time constraint, and its transportation costs are generalized trapezoidal fuzzy numbers. Chanas and Kuchta **[4]** proposed a concept of the optimal solution of the transportation problem with fuzzy cost coefficients and an algorithm determining this solution.

Pramanik and Roy **[18]** showed how the concept of Euclidean distance can be used for modeling MOTP with fuzzy parameters and solving them efficiently using priority based fuzzy goal programming **[25]** under a priority structure to arrive at the most satisfactory decision in the decision making environment, on the basis of the needs and desires of the decision making unit.

2. The Formulation of MOLTP with Fuzzy Parameters

The MOLTP **[1]** is formulated as follows:

min $\quad z^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k x_{ij}, \; k = 1,2,\ldots,K$

Subject to

$$\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_i && i = 1,2 \ldots\ldots m \\ &\sum_{i=1}^{m} x_{ij} = b_j && j = 1,2 \ldots\ldots n \\ &x_{ij} \geq 0, \; \forall i \text{ and } j && \end{aligned} \qquad (1)$$

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Where x_{ij} is decision variable refers to product quantity that transported from supply point *i* to demand point *j*. The capacities of the supply and demand points are denoted by a_i and b_j.

The unit cost for transporting the goods from supply point *i* to demand point *j* for the objective *k*, (*k* = 1, 2. . . *K*) is denoted by C_{ij}^k where *K* is the number of the objective functions. If at least one of these parameters is assumed as fuzzy, then a MOLTP with Fuzzy Parameters arises. In such case $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$, and $\tilde{b}_1, \tilde{b}_2, \ldots \tilde{b}_m$ are called as *n* fuzzy supply and *m* fuzzy demand quantities, respectively. Similarly, $\tilde{C}_{ij}^k$ is called as fuzzy unit transportation cost from supply point *i* to demand point *j* for the objective *k*, (*k* = 1, 2. . . *K*) .

3. Preliminaries

Definition: 3.1.

Pareto-optimal solution for MOLTP [8]

Let S be the feasible region of (1). $\mathbf{x}^* \in \mathbf{S}$ is said to be a Pareto-optimal solution (strongly efficient or non-dominated) if and only if there does not exist another $\mathbf{x} \in \mathbf{S}$ such that $z^k(x) \leq z^k(\mathbf{x}^*)$ for all k = 1, 2, . . . , K and $z^k(x) \neq z^k(\mathbf{x}^*)$ for at least one k = 1, 2, . . . , K.

Definition: 3.2.

Compromise solution for MOLTP

A feasible solution $\mathbf{x}^* \in \mathbf{S}$ is called a compromise solution of (1) if and only if $\mathbf{x}^* \in \mathbf{E}$ and $z^k(\mathbf{x}^*) \leq \bigwedge_{\mathbf{x} \in \mathbf{S}} z(\mathbf{x})$, where $z(x) = (z^1(x), z^2(x), \ldots, z^k(x))$, $\bigwedge$ stands for "min" operator and E is the set of Pareto-optimal solutions.

Definition: 3. 3.

Trapezoidal Fuzzy Number:

(TFN) is a convex fuzzy set which is defined as $\tilde{A} = (x, \mu_{\tilde{A}_i}(x))$ where:

$$\mu_{\tilde{A}_i}(x) = \begin{cases} 0 & , x \leq a_i^1 \\ \dfrac{(x - a_i^1)}{(a_i^2 - a_i^1)}, & a_i^1 < x \leq a_i^2 \\ 1 \;, & a_i^2 < x \leq a_i^3 \\ \dfrac{(a_i^4 - x)}{(a_i^4 - a_i^3)} & , a_i^3 < x \leq a_i^4 \\ 0 & , x > a_i^4 \end{cases}$$

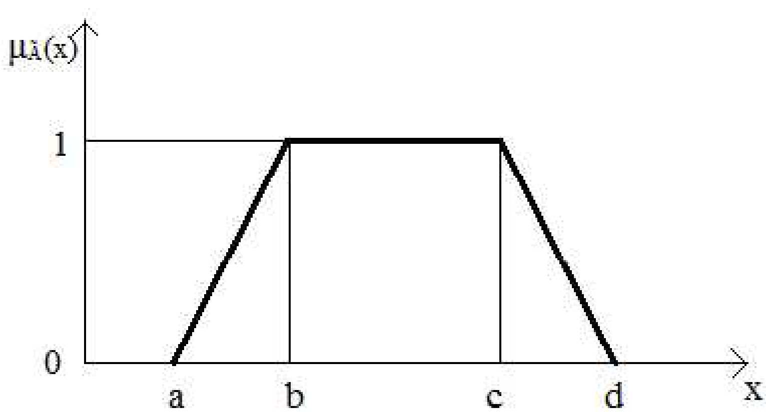

Fig.1 Trapezoidal Fuzzy Number (TFN)

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4. Compensatory Fuzzy Aggregation Operators

There are several fuzzy aggregation operators. The detailed information about them exists in **[26]** and **[21]**. The most important aspect in the fuzzy approach is the compensatory or non-compensatory nature of the aggregation operator. Several investigators **[11, 15, 19, 26]** have discussed this aspect.

Using the linear membership function, Zimmermann **[27]** proposed the "min" operator model to the Multi-objective linear problem (MOLP). It is usually used due to its easy computation. Although the "min" operator method has been proven to have several nice properties **[15]**, the solution generated by min operator does not guarantee compensatory and Pareto-optimality **[7, 13, 24]**. The disadvantage of the aggregation operator "min" is that it is non-compensatory. In other words, the results obtained by the "min" operator represent the worst situation and cannot be compensated by other members which may be very good. On the other hand, the decision modeled with average operator is called fully compensatory in the sense that it maximizes the arithmetic mean value of all membership functions. Zimmermann and Zysno **[28]** show that most of the decisions taken in the real world are neither non-compensatory (min operator) nor fully compensatory and suggested a class of hybrid compensatory operators with γ compensation parameter. Basing on the γ-operator, Werners **[23]** introduced the compensatory "fuzzy and" operator which is the convex combinations of min and arithmetical mean:

$\mu_{and} = \gamma \min_i \mu_i + \frac{(1-\gamma)}{m}(\sum_i \mu_i)$,

where $0 \leq \mu_i \leq 1$, $i = 1, 2, \ldots, m$ and the magnitude of $\gamma \in [0,1]$ represent the grade of compensation.

This operator is not inductive and associative, it is commutative, idempotent, strictly monotonic increasing in each component, continuous and compensatory. In literature, it is showed that the solution generated by Werners' compensatory "fuzzy and" operator does guarantee compensatory and Pareto-optimality for MOLP **[15, 19–24, 28]**. Thus this operator is also suitable for our MOLTP. Therefore, due to its advantages, in this paper, we used Werner's compensatory "fuzzy and" operator.

5. A Compensatory Fuzzy Approach to MOLTP with Fuzzy Parameters [8]

Our compensatory fuzzy approach aims to convert the fuzzy supply and demand quantities to crisp ones. First of all, the fuzzy supply and demand quantities are converted to crisp forms to satisfy the balance condition. By using of the "min" fuzzy operator model proposed by Zimmermann **[27],** the problem

max min $\mu a_i, \mu b_j$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots,$

subject to

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 0$$

$$\sum_{i=1}^{n} x_{ij} = b_j, \text{ j=1, 2...m}$$

$$\forall a_i, b_j \geq 0, \quad i=1, 2\ldots\ldots m, j=1,2\ldots,n$$

is solved for obtaining a solution which maximizes the least degree of satisfaction among all supply and demand quantities. By introducing the auxiliary variable β,

$$\min \mu a_i, \mu b_j = \beta \quad \Rightarrow \mu a_i \geq \beta, \mu b_j \geq \beta,$$

this problem can be converted into the following equivalent maximization problem:

$$\begin{aligned} &\max \beta \\ &\text{subject to } \mu a_i(a_i) \geq \beta, \ i=1,2,\ldots..m \\ &\mu b_j(b_j) \geq \beta, \ j=1,2,\ldots..m \\ &\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = 0 \qquad (1) \\ &\forall a_i, b_j \geq 0, \quad i=1,2\ldots\ldots m, \ j=1,2\ldots,n \\ &\beta \in [0,1] \end{aligned}$$

By solving (1), crisp supply-demand quantities are determined at the maximum satisfactory degree $\bar{\beta}$ in order to get the following balance condition:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Let L_k and U_k be the lower and upper bounds of the objective function z^k, respectively. L_k and U_k can be determined as follows: Solve the MOLTP as a single objective TP using each time only one objective and ignoring all others. Determine the corresponding values for every objective at each solution derived. And find the best (L_k) and the worst (U_k) values corresponding to the set of solutions.

The lower and upper bounds L_k and U_k can also be determined for each objective $z^k(x)(k = 1, 2, \ldots, K)$ as follows:

$$L_k = \min_{x\in S} z^k(x),\quad U_k = \max_{x\in S} z^k(x)$$

where S is the feasible solution space that is satisfied supply-demand and non-negativity constraints. For the sake of simplicity, we used the linear membership function:

$$\mu_k(z_k) = \begin{cases} 1 & if \quad Z_k < L_k \\ \frac{U_k - Z_k}{U_k - L_k} & if \quad L_k \le Z_k \le U_k \\ 0 & if \quad Z_k \ge U_k \end{cases}$$

Here, $L_k \neq U_k,\ k = 1,2 \ldots K$ *and in the case of* $L_k = U_k,\ \mu_k(z_k(x)) = 1$. The membership function $\mu_k(z_k)$ is linear and strictly monotone decreasing for Z^k in the interval [L_k, U_k].Using Zimmermann's minimum operator **[27]**, MOLTP can be written as:

$$\max_x \min_k \quad \mu_k(z_k(x)) \qquad (2)$$

Subject to $x \in S$

By introducing an auxiliary variable λ, (2) can be transformed into the following equivalent conventional LP problem:

$$\begin{aligned} &\text{Max} \quad \lambda \\ &\text{Subject to } \mu_k(z_k(x)) \ge \lambda, \quad k = 1,2 \ldots K \\ &x \in S \qquad (3) \\ &\lambda \in [0,1] \end{aligned}$$

It is pointed out that Zimmermann's min operator model doesn't always yield a strongly - efficient solution **[7, 13, 24]**. By using Werner's operator,**[13]** is converted to as follows:

$$\begin{aligned} &\text{Max} \quad \mu_{and} = \lambda + \frac{(1-\gamma)}{K}(\lambda_1 + \lambda_2 + \cdots + \lambda_k) \\ &\text{Subject to } x \in S \\ &\mu_k(z_k(x)) \ge \lambda + \lambda_k, \quad k = 1,2 \ldots K \\ &\lambda + \lambda_k \le 1, \ k = 1,2 \ldots K \qquad (4) \\ &\gamma \in [0,1] \end{aligned}$$

So, our compensatory model generates compensatory compromise Pareto-optimal solutions for MOLTP.

We shall prove this assertion in the following theorem.

Theorem: 1.

If (x, λ^x) is an optimal solution of problem (4), then x is a Pareto-optimal solution for MOLTP, where $\lambda^x = (\lambda^x, \lambda_1^x, \lambda_2^x, \ldots, \lambda_k^x)$

Proof: Suppose, to the contrary, there exists a feasible solution (y, λ^y) such that y $\succ x$.

This means $z_k(y) \le z_k(x)$, k = 1, 2, . . . , K, and $z_k(y) < z_k(x)$ for some k. Thus, for the membership functions of objectives, it can be written as

$\mu_k(z_k(y)) \geq \mu_k(z_k(x)), \forall k = 1, 2, \ldots, K$ and $\mu_k(z_k(y)) > \mu_k(z_k(x))$,

for some k. And this implies that there exist λ_k^x and λ_k^y satisfying

$\mu_k(z_k(y)) = \lambda^* + \lambda_k^y \geq \mu_k(z_k(x)) = \lambda^* + \lambda_k^x$, $\forall k = 1, 2, \ldots, K$

and

$\mu_k(z_k(y)) = \lambda^* + \lambda_k^y > \mu_k(z_k(x)) = \lambda^* + \lambda_k^x$, for some k. Therefore, it holds that

$\lambda_k^y \geq \lambda_k^x$, $\forall$ k = 1, 2, . . . , K and $\lambda_k^y > \lambda_k^x$, for some k.

This means that $\sum_{k=1}^{K} \lambda_k^y > \sum_{k=1}^{K} \lambda_k^x$ and so

$$\mu_{and}(y) = \lambda^* + \frac{(1-\gamma)}{K}\left(\sum_{k=1}^{K} \lambda_k^y\right) > \lambda^* + \frac{(1-\gamma)}{K}\left(\sum_{k=1}^{K} \lambda_k^x\right) = \mu_{and}(x)$$

that is, $\mu_{and}(y, \lambda^y) > \mu_{and}(x, \lambda^x)$, and this is contradictory to the fact that (x, λ^x) is an optimal solution to problem (4). If required, Pareto-optimality test **[2]** can be applied to the solutions of (4) and it could be seen that these solutions are Pareto-optimal for MOLTP.

6. Linear Programming Formulation of MOTP [16]

A MOTP can be stated as:

Minimize $Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^k x_{ij}$, $k = 1,2,...,K$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2 \ldots \ldots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1, 2 \ldots \ldots n$$

$$x_{ij} \geq 0, \ \forall i \text{ and } j$$

The subscription Z_k and superscript on C_{ij}^k denote the K^{th} penalty criterion. We assume that $a_i \geq 0$ for all *i*, $b_j \geq 0$ for all j, $C_{ij} \geq 0$ for all *i* and *j*, and

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (Equilibrium condition)

a_i is the quantity of material available at source

O_i (i=1, 2 ,...,m)

b_j is the quantity of material required at destination

D_j (j=1 , 2 ,..., n) and

C_{ij} is fuzzy unit cost of transportation from source O_i to destination D_j.

7. Fuzzy Programming Technique to Solve MOTP

To solve MOTP in Fuzzy Programming Technique **[12]** , we first find the lower bound as L_k and the upper bound as U_k for the K^{th} objective function Z_k, k =1, 2,. ...,k where U_k is the highest acceptable level of achievement for objective k, L_k *is* the aspired level of achievement for objective k and $d_k = U_k - L_k$ *is* the degradation allowance for objective k.

To form a fuzzy model, convert it into a crisp model with the aspiration levels for each of the objective are to be specified. The solution of MOTP can be obtained by the following steps:

Step: 1.Solve the MOTP as a single-objective transportation problem K times by taking one of the objectives at a time

Step: 2. Determine the corresponding values for every objective at each solution derived. According to each solution and value for every objective, we can find a pay-off matrix as follows:

	$Z_1(X)$	$Z_2(X)$	$Z_K(X)$
$X^{(1)}$	Z_{11}	Z_{22}	Z_{1K}
$X^{(2)}$	Z_{21}	Z_{22}	Z_{2K}
$X^{(K)}$	Z_{k1}	Z_{k2}	Z_{kK}

Where $X^{(1)}$, $X^{(2)}$,…,$X^{(k)}$ are the isolated optimal solutions of the *K* different transportation problems for K different objective function, $Z_{ij}=Z_j(X^i)$, $i=1,2,\ldots,K;\ j = 1, 2,\ldots,k$ be the i^{th} row and j^{th} column element of the pay-off matrix.

Step: 3. From step 2, find for each objective the U_k and the L_k corresponding to the set of solution, where, U_k = maximum($Z_{1k}, Z_{2k}, \ldots Z_{kk}$) and

L_k = minimum $(Z_{1k}, Z2k, \ldots, Z_{kk})\ k = 1,2,\ldots,K$

An initial fuzzy model of the problem can be obtained as

X_{ij}, $i=1, 2\ldots m,\ j = 1, 2\ldots n$

$Z_k \le L_k$, k =1, 2… k

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2\ldots\ldots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2\ldots\ldots n$$

$$x_{ij} \ge 0, \ \forall i \text{ and } j$$

Step: 4. Define a membership function $\mu(Z_k)$ for the k^{th} objective function

Step: 5. Convert the fuzzy mode of the problem, obtained in step, into the following crisp Model;

Maximize λ

Subject to $\lambda \le \mu(Z_k)$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2\ldots\ldots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2\ldots\ldots n$$

$$x_{ij} \ge 0, \ \forall i \text{ and } j$$

$$\lambda \ge 0$$

Step: 6. Solve the crisp model by an appropriate mathematical programming algorithm

Step: 7 The solution obtained in step 6 will be the optimal compromise solution of the MOTP

8. Fuzzy Programming Technique with Hyperbolic Membership Function [17]

A hyperbolic membership functions is defined by

$$\mu^H(Z_k) = \begin{cases} 1 & if \quad Z_k \le L_k \\ \frac{1}{2}\frac{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} - e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}}{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} + e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}} + \frac{1}{2} & if \quad L_k < F^k(x) < U_k \\ 0 & if \quad Z_k \ge U_k \end{cases}$$

Where $a_k = \frac{6}{(U_k - L_k)}$

If we will use the hyperbolic membership function then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

Subject to

$$\lambda \le \frac{1}{2}\frac{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} - e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}}{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} + e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}} + \frac{1}{2}, \ k = 1,2,\ldots k \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2\ldots\ldots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2\ldots\ldots n \qquad (2)$$
$$x_{ij} \ge 0, \ , \ i \text{ and } j$$

$\lambda \geq 0$

Constraint (1) can further be simplified as:

$$\lambda \leq \frac{1}{2}\tanh\left[\left\{\frac{U_k+L_k}{2} - Z_k(X)\right\}u_k\right] + \frac{1}{2}$$

$$2\lambda \leq \tanh\left[\left\{\frac{U_k+L_k}{2} - Z_k(X)\right\}u_k\right] + 1$$

$$tanh^{-1}(2\lambda - 1) \leq \left\{\frac{U_k+L_k}{2} - Z_k(X)\right\}u_k$$

$$a_k Z_k + tanh^{-1}(2\lambda - 1) \leq \frac{(U_k+L_k)u_k}{2}$$

Now, putting $tanh^{-1}(2\lambda - 1) = X_{mn+1}$, constraint (1) is converted to

$$a_k Z_k(X) + X_{mn+1} \leq \frac{(U_k + L_k)a_k}{2}$$

Hence, the given problem is simplified as:

Maximize X_{mn+1}

Subject to

$a_k Z_k(X) + X_{mn+1} \leq \frac{(U_k+L_k)u_k}{2}$, k=1,2,….k

$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,2 \ldots\ldots m$

$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2 \ldots\ldots n \qquad (2)$

$x_{ij} \geq 0$, , i and j

$X_{mn+1} \geq 0$, where $X_{mn+1} = tanh^{-1}(2\lambda - 1)$

7. Example

Let us consider a MOLTP with the following characteristics:

Supplies: $\tilde{a_1}$ = (16,25,36,49), $\tilde{a_2}$ = (36,49,64,81), $\tilde{a_3}$ = (25,36,49,64),

Demands: $\tilde{b_1}$ = (36,49,64,81), $\tilde{b_2}$ = (37,45,60,77), $\tilde{b_3}$ = (4,16,25,36)

Costs:

$$\tilde{C_1} = \begin{bmatrix} (2,4,6,9) & (8,10,12,14) & (10,12,14,16) \\ (3,5,7,10) & (4,6,8,12) & (9,10,12,15) \\ (11,14,16,18) & (13,15,18,20) & (6,8,10,12) \end{bmatrix}$$

$$\tilde{C_2} = \begin{bmatrix} (5,8,10,12) & (6,8,10,12) & (12,15,18,20) \\ (7,9,11,14) & (4,6,9,11) & (6,9,12,14) \\ (13,15,18,21) & (15,18,20,22) & (14,16,18,22) \end{bmatrix}$$

Solution:

Using the Robust ranking technique,

$\tilde{a_1}$ = 31.5, $\tilde{a_2}$ = 57.5, $\tilde{a_3}$ = 43.5,

$\tilde{b_1}$ = 57.5, $\tilde{b_2}$ = 54.75, $\tilde{b_3}$ = 20.25

Costs:

$$\tilde{C_1} = \begin{bmatrix} 5.25 & 11 & 13 \\ 6.25 & 7.5 & 11.5 \\ 14.75 & 16.5 & 9 \end{bmatrix}$$

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$$\widetilde{C_2} = \begin{bmatrix} 8.75 & 9 & 16.25 \\ 10.25 & 7.5 & 10.25 \\ 16.75 & 18.75 & 17.5 \end{bmatrix}$$

The developed problem can be formulated as:

Min $Z_1 = 5.25X_{11}+11X_{12}+13X_{13}+6.25X_{21}+7.5X_{22}+11.5X_{23}+$

$14.75X_{31}+16.5X_{32}+9X_{33}$ (2)

Min $Z_2 = 8.75X_{11}+9X_{12}+16.25X_{13}+10.25X_{21}+7.5X_{22}+10.25X_{23}+$

$16.75X_{31}+18.75X_{32}+17.5X_{33}$ (3)

Subject to:

$\sum_{j=1}^{3} X_{1j} = 31.5, \sum_{j=1}^{3} X_{2j} = 57.5, \sum_{j=1}^{3} X_{3j} = 43.5$

$\sum_{i=1}^{3} X_{i1} = 57.5, \sum_{i=1}^{3} X_{i2} = 54.75, \sum_{i=1}^{3} X_{i3} = 20.25$ (4)

$X_{ij} \geq 0, i = 1,2,3; j = 1,2,3$

Here Z_1 and Z_2 represent the total cost and the total time of transportation respectively.

Solving equations (2) and (4), we obtain the optimal solution as:

$X^1 = \{X_{11} = 31.5, X_{21} = 2.75, X_{22} = 54.75, X_{31} = 23.25, X_{33} = 20.25\}$

$Z_1(X^1) = 1118.375$, $Z_2(X^1) = 1458.25$

Solving equations (3) and (4), we obtain the optimal solution as:

$X^2 = \{X_{11} = 14, X_{12} = 17.5, X_{21} = 20.25, X_{22} = 37.25, X_{31} = 23.25; X_{33} = 20.25\}$

$Z_1(X^2) = 1197.125$, $Z_2(X^2) = 1510.75$

The outcomes obtained from step 1 give the following pay-off matrix as;

$$\begin{array}{c} \\ X^1 \\ X^2 \end{array} \begin{array}{cc} Z_1(X^i) & Z_2(X^i) \\ \left(1118.375 \right. & \left. 1458.25 \right) \\ \left(1197.125 \right. & \left. 1510.75 \right) \end{array}$$

From the pay – off matrix we obtain

$U_1 = maximum\{1118.375, 1197.125\} = 1197.125$

$L_1 = maximum\{1118.375, 1197.125\} = 1118.375$

$U_2 = maximum\{1458.25, 1510.75\} = 1510.75$

$L_2 = maximum\{1458.25, 1510.75\} = 1458.25$

If a compensatory fuzzy approach is employed, the crisp model can be presented as follows:

Max $\mu_{and} = \lambda + \frac{(1-\gamma)}{K}(\lambda_1 + \lambda_2 + \cdots + \lambda_k)$

Subject to

$5.25X_{11}+11X_{12}+13X_{13}+6.25X_{21}+7.5X_{22}+11.5X_{23}+$

$14.75X_{31}+16.5X_{32}+9X_{33}+78.75\lambda+78.75\lambda_1 \le 1197.125$

$8.75X_{11}+9X_{12}+16.25X_{13}+10.25X_{21}+7.5X_{22}+10.25X_{23}+$

$16.75X_{31}+18.75X_{32}+17.5X_{33}+52.5\lambda+52.5\lambda_2 \le 1510.75$

$\sum_{j=1}^{3} X_{1j} = 31.5, \sum_{j=1}^{3} X_{2j} = 57.5, \sum_{j=1}^{3} X_{3j} = 43.5$

$\sum_{i=1}^{3} X_{i1} = 57.5,$ $\sum_{i=1}^{3} X_{i2} = 54.75, \sum_{i=1}^{3} X_{i3} = 20.25$

$\lambda+\lambda_1 \le 1,\ \lambda+\lambda_2 \le 1,$

$\lambda, \lambda_1, \lambda_2, \gamma \in [0,1]$

$X_{ij} \ge 0, i = 1,2,3; j = 1,2,3$

The compensatory compromise pareto - optimal solution of the above problem is thus presented as below:

$X^* = \{X_{11} = 31.5, X_{21} = 2.75, X_{31} = 23.25, X_{22} = 54.75, X_{33} = 20.25\}$

$Z_1^* = 1118.375;\ Z_1^* = 1458.25;\ \mu_{and} = 1;$

If we use the hyperbolic membership functions, an equivalent crisp model can be formulated as:

Maximize X10
Subject to:
$0.4001X_{11}+0.8382X_{12}+0.9906X_{13}+0.4763X_{21}+0.5715X_{22}+0.8763X_{23}+$

$1.12395X_{31}+1.2573X_{32}+0.6858X_{33}+X_{10} \le 88.22$

$1.0001X_{11}+1.0287X_{12}+1.8574X_{13}+1.1716X_{21}+0.8573X_{22}+1.1716X_{23}+$

$1.9145X_{31}+2.1431X_{32}+2.0003X_{33}+X_{10} \le 169.68$

$\sum_{j=1}^{3} X_{1j} = 31.5, \sum_{j=1}^{3} X_{2j} = 57.5, \sum_{j=1}^{3} X_{3j} = 43.5$

$\sum_{i=1}^{3} X_{i1} = 57.5, \sum_{i=1}^{3} X_{i2} = 54.75, \sum_{i=1}^{3} X_{i3} = 20.25$

$X_{ij} \ge 0, i = 1,2,3; j = 1,2,3$

Solving the above problem, the optimal solution is shown a follows:
$X^* = \{X_{11} = 2.75, X_{21} = 31.5, X_{22} = 54.75, X_{31} = 20.25, X_{33} = 23.25\}$

$Z_1^* = 1129.875;\ Z_1^* = 1503.625;\ X_{10} = 14$

8. Conclusion: In this paper, we deal with MOLTP whose costs and supply-demand quantities are given as trapezoidal fuzzy numbers. In the first stage the defuzzification of the parameters of MOLTP is done. Applying Werner's μ_{and} operator, our approach generates a solution for each member of this set. This compromise solution of MOLTP with fuzzy parameters is both compensatory and Pareto-optimal.

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